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AN ECO-EPIDEMIOLOGICAL MODEL OF BANANA DISEASE WITH THE IMPACT OF CULTURAL CONTROLS AND MEASURING DISEASE CAUSES

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ABSTRACT

This paper discusses a mathematical model for Panama wilt with cultural controls and the model is constructed since the two major causes of the disease. Disease-free and endemic equilibrium points were identified. The reproduction number is calculated via the next-generation matrix. The local and global stability of the equilibrium points are studied. Bifurcation analysis is carried out. Numerical simulations are performed in MATLAB.

Keywords: Global stability, Bifurcation, Equilibrium points, Panama wilt, Cultural controls

1. INTRODUCTION

The existence of living and nonliving things in the world is determined by their relationships with each other. On a larger scale, living things include humans, animals, and plants. In addition, humans are always dependent on plants for oxygen, food, etc. **[1]**. Therefore, plants are considered essential for human existence. This process involves consuming plant pro ducts such as vegetables, fruits, and leaves, which are rich in proteins, vitamins, minerals, fiber, etc. There are two types of plants consumed by humans. They can be used as food or for medicinal purposes. Some plants serve two purposes a nd are therefore called healing foods.

Among many plants banana is a medicinal plant in which all parts are used for specific purposes. For example, the fruit is eaten as a diet food, the flower helps trea t breast cancer, and diabetes, and maintain the menstrual cycle, and banana leaves are used as dishes in countries such as South India and Thailand. Banana sprouts balance blood sugar levels and are good for kidneys , etc. Bananas are considered a favorite food for many people around the world. For these reasons, bananas are grown in many countries such as India, Brazil, Ecuador, China, the Philippines, Indonesia, Costa Rica, Mexico, Thailand, and Colombia. Among them, India is the largest producer of bananas which are also economically important crops.

Banana plants are affected by many pathogens such as fungi, bacteria, and viruses, and diseases such as airborne diseases, and soil diseases. The most destructive disease of bananas in all seasons is Panama wilt or Fusarium wilt. The pathogen causing this disease is *Fusarium oxysporum* f. sp. Cubence (Foc) **[3, 5]**. It is a vascular wilt disease that affects the woody part of the vascular tissue, causing discolor ation and wilting of leaves **[5, 12]**.

There are two types of symptoms.

- External symptoms: The most common symptom is the yellow browning of leaves. This starts at the margin of the leaves and becomes yellow, and at first, it looks like a potassium deficiency, after which it spreads all over the leaf. Eventually, the leaf becomes bright yellow in color. This result in leaves wilt, and then the plants die. It spreads to all the leaves in the plant and the wilt of the leaves appears to be a brown skirt.
- Internal symptoms: Internal symptoms are severe. It affects the xylem vessels. This forms a fungus in the stem tissues, which is reddish-brown in color. These discolors are observed only when the plant is cut longitudinally. This may produce infected suckers but the fruit does not necessarily show any symptoms **[14]**.

During the disease cycle, infection initially occurs with the fungus *Fusarium oxysporu m* F. sp. Cubence (Foc), and plants next to infected plants are also af -fected by this fungus. It can be spread through tools used for cleaning. Climate ch anges such as temperature, storm damage and wet or dry conditions (humidity) also af -fect the spread of the disease to other plants **[12]**.

Disease transmission was studied via a mathematical model in the 19th century. A mathematical model assists in detecting the transmission of diseases. The mathematical model has become a powerful tool that acknowledges the influences of transmittable disease **[7]**. Recently, mathematical models have been developed for the analysis of controls with diseases. The diseases that cause diseases worldwide have been analyzed on the basis of outbreak and deterioration via mathematical models and computational mechanisms **[9]**. In accordance with computational modeling, many appease strategies have been developed for these models. The study of plant disease is known as epidemiology. Many researchers, epidemiologists and mathematicians are interested in predicting disease spread and control via mathematical models. Academicians have built an epidemiological model that intimately reflects the relationship of disease transmission from one to another. The SIR model is used for infectious diseases in susceptible (S), infected (I) and recovered (R) populations **[17]**. Many researchers subsequently developed and improved the SIR model according to the disease. The model is explained via ordinary differential equations.

B. Nanyonga *et al*. studied a mathematical model, for contaminated tools that help in the spread of disease. He used the Runge–Kutta algorithm to identify the spread of the disease **[18]**. Juliet Nakakawa *et al*. investigated a model with controls that addresses debudding and roughening. Researchers have analyzed how these controls help limit disease in mixed cultivation **[10]**. He also built a model to study disease spread through vertical transmission **[11]**.

Kweynaga Eliab Horub *et al*. analyzed a host-vector model for Xanthomonas wilt in bananas **[13]**. Eliab Horub J *et al*. built a new mathematical model with asymptomatic and symptomatic stages of disease **[6]**. Giovanni Bubici studied fusarium wilt and how to control fusarium wilt via biological agents **[8]**. Muhammed Akmal Zawawi *et al*. performed a statistical analysis of the soil properties associated with Fusarium wilt and its economic importance for banana production in many countries **[13]**. Sanburi Tansah Tresna *et al*. investigated the transmission of soybean mosaic disease with aphid interventions and photo periodicity **[20, 21]**. In the following sections, we analyzed Fusarium wilt via cultural controls to curb the disease.

2 MATHEMATICAL MODEL

In this section, a mathematical model is formulated via ordinary differential equations. The system of equations is given below:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda + \big(\gamma_1 + \gamma_3 + \alpha - (\phi + \rho)\big)S - (\pi_1 + \pi_2)\beta_1 S I_0 \\
\frac{dI_0}{dt} &= (\pi_1 + \pi_2)\beta_1 S I_0 - (\pi_1 + \pi_2)\beta_2 I_0 - (\gamma_2 + \omega_2) I_0 \\
\frac{dI_1}{dt} &= (\pi_1 + \pi_2)\beta_2 I_0 - (\gamma_2 + \omega_3) I_1 \\
\frac{dN}{dt} &= \rho S - \omega_1 N \\
\frac{dP}{dt} &= \omega_1 N + \omega_2 I_0 + \omega_3 I_1 - hP
\end{aligned}\right\} \tag{1}$$

Where $\phi + \rho = \alpha;\ \pi_1 + \pi_2 = b$

The model's graphical representation is shown in **Figure 1**.

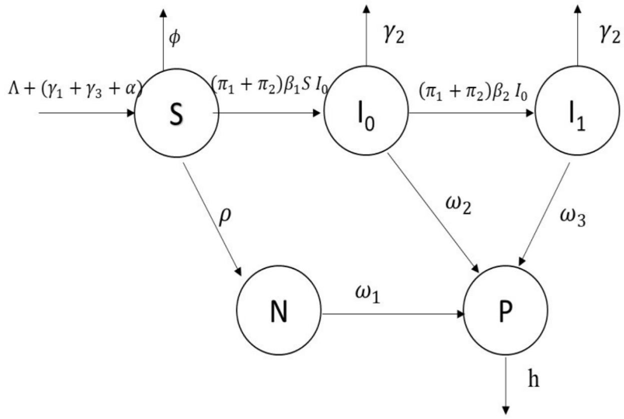

Figure 1: Model's pictorial representation

Table 1: Parameter descriptions

Parameters	Representation	Parameters	Representation
Λ	**Sampling rate**	π_1	**Humidity**
π_2	**Temperature and climate change**	α	**Suckers rate**
φ	**Degeneration rate**	ρ	**Disinfected plant rate**
β_1	**Initial infection of disease**	β_2	**Severe infection of disease**
γ_1	**Proper drainage**	γ_2	**Removing the infected crop**
γ_3	**Intercropping, to avoid and reduce infection**	ω_1	**High and healthy production of fruits**
ω_2	**Low production of fruits**	ω_3	**Unhealthy production of fruits**
h	**Total harvest**		

The model describes how cultural controls help prevent and prevent the spread of the disease in the field. The model is built with five populations (i) susceptible (S), (ii) initial infection (I_0),
(iii) severe infection (I_1), (iv) non diseased(N) and (v) fruit production (P) plants.

Here the assumptions are The susceptible plants are considered to have a seedling rate (Λ) and a sucker rate (α). While the plants were being planted, we added cultural controls such as proper drainage (γ_1) and intercropping (γ_3) to avoid infection. Here we have two different stages if the plant becomes infected, which may be the initial infection (β_1), leading to secondary infection (β_2) otherwise the plant will not be infected (ρ), resulting in healthy fruits(ω_1).

- The initial infection occurs when the plants encounter the fungus. This causes initial infection(β_1) in the plant and the infection may be caused by humidity(π_1) and temperature (π_2). Some plants may not resist infection, which can lead to severe infection., i.e., secondary infection(β_2). Therefore, plants may result in low fruit production from initially infected plants (ω_2) and yield unhealthy fruit production from severely infected plants (ω_3).

- If vulnerable plants are not infected and produce only good, healthy fruits, they may lead to non disease outcomes.
- Fruits produced from various states are represented in the previous assumption, and the total harvest is taken as 'h'.

2.1 Existence and uniqueness of the solution

Theorem 2.1.

If we assume that $R(Y)$ is a function of S, I_0, I_1, N and P, the initial condition is $Y_0 = (S_0, I_{00}, I_{10}, N_0, P_0) \in R_j(Y), j = 1,2,3,4,5$. Then, for all Y_0, $\|R(Y) - R(\bar{Y})\| \leq L|Y - \bar{Y}|$ satisfies the unique solution $Y(t) \in R_j$, where L is the Lipchitz constant.

Proof. Let us consider R_j as follows:

$$
\begin{aligned}
R_1(Y) &= \Lambda + (\gamma_1 + \gamma_3 + \alpha - (\varphi + \rho))S - (\pi_1 + \pi_2)\beta_1 S I_0 \\
R_2(Y) &= (\pi_1 + \pi_2)\beta_1 S I_0 - (\pi_1 + \pi_2)\beta_2 I_0 - (\gamma_2 + \omega_2) I_0 \\
R_3(Y) &= (\pi_1 + \pi_2)\beta_2 I_0 - \gamma_2 I_1 - \omega_3 I_1 \\
R_4(Y) &= \rho S - \omega_1 N \\
R_5(Y) &= \omega_1 N + \omega_2 I_0 + \omega_3 I_1 - hP
\end{aligned} \tag{2}
$$

Rewriting (2) as

$$
\begin{aligned}
R_1(Y) &= \Lambda + AS - B\beta_1 S I_0 \\
R_2(Y) &= B\beta_1 S I_0 - B\beta_2 I_0 - C I_0 \\
R_3(Y) &= B\beta_2 I_0 - D I_1 \\
R_4(Y) &= \rho S - \omega_1 N \\
R_5(Y) &= \omega_1 N + \omega_2 I_0 + \omega_3 I_1 - hP
\end{aligned} \tag{3}
$$

Here

$$
\begin{aligned}
A &= (\gamma_1 + \gamma_3 + \alpha - (\varphi + \rho)) \\
B &= (\pi_1 + \pi_2) \\
C &= \gamma_2 + \omega_2 \\
D &= \gamma_2 - \omega_3
\end{aligned} \tag{4}
$$

Then

$$
\begin{aligned}
|R(Y) - R(\bar{Y})| &= |(R_1(Y) + R_2(Y) + R_3(Y) + R_4(Y) + R_5(Y)) - (R_1(\bar{Y}) + R_2(\bar{Y}) + R_3(\bar{Y}) + R_4(\bar{Y}) + R_5(\bar{Y}))| \\
&= |(R_1(Y) - R_1(\bar{Y})| + |R_2(Y) - R_2(\bar{Y})| + |R_3(Y) - R_3(\bar{Y})| + |R_4(Y) - R_4(\bar{Y})| + |R_5(Y) - R_5(\bar{Y})| \\
&\leq |\Lambda + AS - B\beta_1 S I_0 - \Lambda - A\bar{S} + B\beta_1 \bar{S}\bar{I}_0| + |B\beta_1 S I_0 - B\beta_2 I_0 - C I_0| \\
&\quad + |B\beta_2 I_0 - D I_1 - B\beta_2 \bar{I}_0 + D\bar{I}_1 + |\rho S - \omega_1 N - \rho\bar{S} + \omega_1 \bar{N}| \\
&\quad + |\omega_1 N + \omega_2 I_0 + \omega_1 I_1 - hP - \omega_1 \bar{N} - \omega_2 \bar{I}_0 - \omega_3 \bar{I}_1 + h\bar{P}| \\
&= (A + 2B\beta_1\eta_1 + \rho)\,|(S - \bar{S})| + (2B\beta_1\eta_2 + 2B\beta_2 + C + \omega_2)|I_0 - \bar{I}_0| \\
&\quad + (D + \omega_3)|I_1 - \bar{I}_1| + 2\omega_1|N - \bar{N}| + h|P - \bar{P}| \\
&\leq L|(S - \bar{S}) + (I_0 - \bar{I}_0) + (I_1 - \bar{I}_1) + (N - \bar{N}) + (P - \bar{P})| \\
&\leq L|(S, I_0, I_1, N, P) - (\bar{S}, \bar{I}_0, \bar{I}_1, \bar{N}, \bar{P})| \\
&\leq L|Y - \bar{Y}|
\end{aligned} \tag{5}
$$

where $L = max\{A + 2B\beta_1\eta_1, 2B\beta_1\eta 2 + 2B\beta_2 + C + \omega_2, D + \rho + \omega_3, 2\omega_1, h\}$. This shows that there exists a unique solution.

3 Model Analysis

3.1 Equilibrium point

The equilibrium points are found in two scenarios: the first involves finding the points without incorporating infections called disease-free equilibrium and the second involves locating the points while accounting for the infections, called endemic equilibrium.

- (1) Disease-free Equilibrium: $E(S, I_0, I_1 N, P)$

$$= \left(\frac{\Lambda}{(a)-(\gamma_1+\gamma_3+\alpha)}, 0, 0, \frac{\rho\Lambda}{\omega_1((a)-(\gamma_1+\gamma_3+\alpha))}, \frac{\rho\Lambda}{h((a)-(\gamma_1+\gamma_3+\alpha)}) \right), \text{S>0, N>0}$$

and P>0 only if $(\varphi + \rho) > \gamma_1 + \gamma_3 + \alpha$.

- (2) Endemic Equilibrium: $E^*(S, I_0, I_1, N, P)$

$$= \left(\frac{(b)\beta_2+\gamma_2+\omega_2}{(b)\beta_1}, \frac{\Lambda}{(b)\beta_2+\gamma_2+\omega_2} + \frac{(\gamma_1+\gamma_3+\alpha)-a}{b\beta_1}, \frac{(b)\beta_2 I_0}{\gamma_2+\omega_3}, \frac{\rho((b)\beta_2+\gamma_2+\omega_2)}{\omega_1 b\beta_1}, \frac{1}{h}\left(\omega_1 N + \left(\omega_2 + \frac{\omega_3(b)\beta_2}{\gamma_2+\omega_3}\right)\right) I_0\right).$$

Here I_0 is positive only if $\gamma_1 + \gamma_3 + \alpha > a$.

3.2 Basic Reproduction Number

The reproduction number is used to find the subsidiary infections of the plant disease. This is calculated via the next-generation matrix **[22]**, i.e., $R_0 = FV^{-1}$. Here F is the infectious value and V^{-1} is the noninfectious value. The basic reproduction number is calculated as follows:

$$F = \begin{pmatrix} b\beta_1 S & 0 \\ 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} b\beta_2 + \gamma_2 + \omega_2 & 0 \\ -(b)\beta_2 & \gamma_2 + \omega_3 \end{pmatrix} \quad (6)$$

Let us find V^{-1},

$$V^{-1} = \frac{1}{(b\beta_2 + \gamma_2 + \omega_2)(\gamma_2 + \omega_3)} \begin{vmatrix} \gamma_2 + \omega_3 & b\beta_2 \\ 0 & b\beta_2 + \gamma_2 + \omega_2 \end{vmatrix}$$

After simplification

$$V^{-1} = \begin{vmatrix} \frac{1}{(b\beta_2 + \gamma_2 + \omega_2)} & \frac{b\beta_2}{(b\beta_2 + \gamma_2 + \omega_2)(\gamma_2 + \omega_3)} \\ 0 & \frac{1}{(\gamma_2 + \omega_3)} \end{vmatrix}$$

$$FV^{-1} = \begin{pmatrix} b\beta_1 S & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{(b\beta_2 + \gamma_2 + \omega_2)} & \frac{b\beta_2}{(b\beta_2 + \gamma_2 + \omega_2)(\gamma_2 + \omega_3)} \\ 0 & \frac{1}{(\gamma_2 + \omega_3)} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} \frac{b\beta_1 S}{(b\beta_2 + \gamma_2 + \omega_2)} & \frac{b^2\beta_2\beta_1 S}{(b\beta_2 + \gamma_2 + \omega_2)(\gamma_2 + \omega_3)} \\ 0 & 0 \end{pmatrix}$$

Hence the basic reproduction number is $R_0 = \frac{b\beta_1 S}{(b\beta_2+\gamma_2+\omega_2)}$. After the disease-free Equilibrium $R_0 = \frac{(\pi_1+\pi_2)\beta_1\Lambda}{(b\beta_2+\gamma_2+\omega_2)(a-(\gamma_1+\gamma_3+\alpha))}$.

3.3 Local Stability of the Model

Theorem 3.1. *The disease-free equilibrium is stable only if* $R_0 < 1$*; otherwise, it is unstable.*

Proof. From system (1), the Jacobian matrix is

$$J = \begin{pmatrix} \gamma_1+\gamma_3+\alpha-a-b\beta_1 I_0 & -b\beta_1 S & 0 & 0 \\ b\beta_1 I_0 & b\beta_2+\gamma_2+\omega_2 & 0 & 0 \\ 0 & b\beta_2 & -(\gamma_2+\omega_3) & 0 \\ \rho & 0 & 0 & -\omega_1 \end{pmatrix} \quad (7)$$

At the disease-free equilibrium point,

$$J(E) = \begin{pmatrix} \gamma_1+\gamma_3+\alpha-a & -b\beta_1 S & 0 & 0 \\ 0 & b\beta_2+\gamma_2+\omega_2 & 0 & 0 \\ 0 & b\beta_2 & -(\gamma_2+\omega_3) & 0 \\ \rho & 0 & 0 & -\omega_1 \end{pmatrix} \quad (8)$$

The characteristic equation is

$$(\gamma_1+\gamma_3+\alpha-\mathrm{a}-\lambda)(b\beta_1 S-((b\beta_2+\gamma_2+\omega_2)-\lambda)(-(\gamma_2+\omega_3)-\lambda)(-\omega_3-\lambda)=0$$

$$(-(\gamma_2+\omega_3)(-\omega_1-\lambda)\{(\gamma_1+\gamma_3+\alpha-a-\lambda)(b\beta_1 S-(b\beta_2+\gamma_2+\omega_2)-\lambda)\}=0 \quad (9)$$

From (10), the two eigenvalues are $\lambda_3=-(\gamma_2+\omega_3)$ and $\lambda_4=-\omega_1$. Here the two eigenvalues are negative. Other eigenvalues are found as follows:

$$(\gamma_1+\gamma_3+\alpha-a-\lambda)(b\beta_1 S-(b\beta_2+\gamma_2+\omega_2)-\lambda)=0 \quad (10)$$

$$(\gamma_1+\gamma_3+\alpha-a)b\beta_1 S-((\gamma_1+\gamma_3+\alpha-a))(b\beta_2+\gamma_2+\omega_2)-\lambda(\gamma_1+\gamma_3+\alpha-a)$$
$$-\lambda a\beta_1 S+\lambda(b\beta_2+\gamma_2+\omega_2)+\lambda^2=0 \quad (11)$$

$$\lambda^2+\lambda(b\beta_2+\gamma_2+\omega_2-b\beta_1 S-(\gamma_1+\gamma_3+\alpha-a))$$
$$+((\gamma_1+\gamma_3+\alpha-a)b\beta_1 S-(\gamma_1+\gamma_3+\alpha-a)(a\beta_2+\gamma_2+\omega_2))=0 \quad (12)$$

Rewriting equation (12) as

$$\lambda^2+A_0\lambda+A_1=0 \quad (13)$$

where

$$A_0=b\beta_2+\gamma_2+\omega_2-b\beta_1 S-(\gamma_1+\gamma_3+\alpha-a)$$
$$A_1=(\gamma_1+\gamma_3+\alpha-a)b\beta_1 S-(\gamma_1+\gamma_3+\alpha-a)(a\beta_2+\gamma_2+\omega_2)$$

Now using the Routh–Hurwitz criteria, $A_0 > 0$ only if $(\gamma_1+\gamma_3+\alpha) > a$ and $A_1 > 0$ only if $R_0 < 1$. This is locally stable at disease-free equilibrium.

Theorem 3.2. *The endemic equilibrium is stable if* $R_0 > 1$*; otherwise, it is unstable.*

Proof:

The Jacobian matrix in (8) with endemic equilibrium points is

$$J(E^*) = \begin{pmatrix} \gamma_1+\gamma_3+\alpha-a-b\beta_1 I_0 & -b\beta_1 S & 0 & 0 \\ b\beta_1 I_0 & b\beta_2+\gamma_2+\omega_2 & 0 & 0 \\ 0 & b\beta_2 & -(\gamma_2+\omega_3) & 0 \\ \rho & 0 & 0 & -\omega_1 \end{pmatrix}$$

Then the characteristic equation is

$$(-(\gamma_2+\omega_3)-\lambda)(-\omega_1-\lambda)\{(\gamma_1+\gamma_3+\alpha-a-b\beta_1 I_0^*-\lambda)(b\beta_1 S^*-(b\beta_2+\gamma_2+\omega_2)-\lambda)+b^2\beta_2^2 S^* I_0^*\}=0 \quad (14)$$

From (14), the two eigenvalues are $\lambda_3 = -(\omega_3 + \gamma_2)$ and $\lambda_4 = -\omega_1$ and are negative. Now, the other roots are

$$(\gamma_1 + \gamma_3 + \alpha - a - b\beta_1 I_0^* - \lambda)(b\beta_1 S^* - (b\beta_2 + \gamma_2 + \omega_2) - \lambda) + b^2\beta_2^2 S^* I_0^* = 0 \quad (15)$$

$$(\gamma_1 + \gamma_3 + \alpha - a)(b\beta_1 S^*) - (\gamma_1 + \gamma_3 + \alpha - a)(b\beta_2 + \gamma_2 + \omega_2) - \lambda(\gamma_1 + \gamma_3 + \alpha - a) -$$
$$b^2\beta_1\beta_2 S^* I_0^* + b\beta_2 I_0^*(b\beta_2 + \gamma_2 + \omega_2 + b\beta_2 I_0^*\lambda - (b\beta_2 + \gamma_2 + \omega_2)\lambda + \lambda^2 + b^2\beta_2^2 S^* I_0^*) = 0 \quad (16)$$

After simplification,

$$\lambda^2 + \lambda\{b\beta_2 + \gamma_2 + \omega_2) - b\beta_1 S^* + b\beta_2 I^* - (\gamma_1 + \gamma_3 + \alpha - a)\} + \{(\gamma_1 + \gamma_3 + \alpha - a)(b\beta_1 S^*) -$$
$$(\gamma_1 + \gamma_3 + \alpha - a)\{b\beta_2 + \gamma_2 + \omega_2) - b^2\beta_1\beta_2 S^* I_0^* + b\beta_2 I_0^*(b\beta_2 + \gamma_2 + \omega_2) + (b)^2\beta_2^2 S^* I_0^* = 0 \quad (17)$$

Rewriting (17) as

$$\lambda^2 + B_0\lambda + B_1 = 0 \quad (18)$$

Here

$$B_0 = \{b\beta_2 + \gamma_2 + \omega_2) - b\beta_1 S^* + b\beta_2 I^* - (\gamma_1 + \gamma_3 + \alpha - a)$$

$$B_1 = (\gamma_1 + \gamma_3 + \alpha - a)(b\beta_1 S^*) - (\gamma_1 + \gamma_3 + \alpha - a)\{b\beta_2 + \gamma_2 + \omega_2) - b^2\beta_1\beta_2 S^* I_0^*$$
$$+ b\beta_2 I_0^*(b\beta_2 + \gamma_2 + \omega_2) + b^2\beta_2^2 S^* I_0^*$$

Now, using the Routh–Hurwitz criteria, $B_0 > 0$ and $B_1 > 0$ only if $R_0 > 1$. This strain is locally stable at endemic equilibrium.

3.4 Global Stability of Endemic Equilibrium

Theorem 3.3. *The endemic equilibrium of system (1) is globally stable if and only if* $\frac{dV}{dt} < 0$.

Proof. Let V(t) be

$$\left(S - S^* \ln\frac{S}{S^*}\right) + \zeta_1\left(I_0 - I_0^* ln\frac{I_0}{I_0^*}\right) + \zeta_2\left(I_1 - I_1^* ln\frac{I_1}{I_1^*}\right) + \zeta_3\left(N - N^* ln\frac{N}{N^*}\right) + \zeta_4\left(P - P^* ln\frac{P}{P^*}\right) \quad (19)$$

Differentiating V(t),

$$\frac{dV}{dt} = \frac{S - S^*}{S}\frac{dS}{dt} + \zeta_1\frac{I_0 - I_0^*}{I_0}\frac{dI_0}{dt} + \zeta_2\frac{I_1 - I_1^*}{I_1}\frac{dI_1}{dt} + \zeta_3\frac{N - N^*}{N}\frac{dN}{dt} + \zeta_4\frac{P - P^*}{P}\frac{dP}{dt}$$

$$\frac{dV}{dt} = \frac{S - S^*}{S}\left(\Lambda + \left(\gamma_1 + \gamma_3 + \alpha - (\phi + \rho)\right)S - (\pi_1 + \pi_2)\beta_1 S I_0\right)$$
$$+\zeta_1\frac{I_0 - I_0^*}{I_0}((\pi_1 + \pi_2)\beta_1 S I_0 - (\pi_1 + \pi_2)\beta_2 I_0 - (\gamma_2 + \omega_2)I_0)$$
$$+\zeta_2\frac{I_1 - I_1^*}{I_1}((\pi_1 + \pi_2)\beta_2 I_0 - (\gamma_2 + \omega_3)I_1) + \zeta_3\frac{N - N^*}{N}(\rho S - \omega_1 N)$$
$$+\zeta_4\frac{P - P^*}{P}(\omega_1 N + \omega_2 I_0 + \omega_3 I_1 - hP)$$

$$\frac{dV}{dt} = (S - S^*)\left(\frac{\Lambda}{S} + \big(\gamma_1 + \gamma_3 + \alpha - (\phi + \rho)\big) - (\pi_1 + \pi_2)\beta_1 I_0\right) + \zeta_1(I_0 - I_0^*)\big((\pi_1 + \pi_2)\beta_1 - (\pi_1 + \pi_2)\beta_2 - (\gamma_2 + \omega_2)\big) + +\zeta_2(I_1 - I_1^*)((\pi_1 + \pi_2)\beta_2 \frac{I_0}{I_1} - (\gamma_2 + \omega_3)) + \zeta_3(N - N^*)(\rho \frac{S}{N} - \omega_1) + \zeta_4(P - P^*)(\omega_1 \frac{N}{P} + \omega_2 \frac{I_0}{P} + \omega_3 \frac{I_1}{P} - h) \quad (20)$$

From (1),

$$\left.\begin{aligned} \frac{-\Lambda}{S^*} &= \big(\gamma_1 + \gamma_3 + \alpha - (\phi + \rho)\big) - (\pi_1 + \pi_2)\beta_1 I_0^* \\ (\pi_1 + \pi_2)\beta_1 S^* &= (\pi_1 + \pi_2)\beta_2 + \big(\gamma_2 + \omega_2\big) \\ \frac{(\pi_1 + \pi_2)\beta_2 I_0^*}{I_1^*} &= (\gamma_2 + \omega_3) \\ \rho \frac{S^*}{N^*} &= \omega_1 \\ \omega_1 \frac{N^*}{P^*} + \omega_2 \frac{I_0^*}{P^*} + \omega_3 \frac{I_1^*}{P^*} &= h \end{aligned}\right\} \quad (21)$$

Substituting (21) into (20), we obtain

$$\frac{dV}{dt} = (S - S^*)\left(\frac{\Lambda}{S} - (\pi_1 + \pi_2)\beta_1 I_0 - \frac{-\Lambda}{S^*} - (\pi_1 + \pi_2)\beta_1 I_0^*\right) + \zeta_1(I_0 - I_0^*)((\pi_1 + \pi_2)\beta_1 S - (\pi_1 + \pi_2)\beta_1 S^*) + \zeta_2(I_1 - I_1^*)\left(\frac{(\pi_1 + \pi_2)\beta_2 I_0}{I_1} - \frac{(\pi_1 + \pi_2)\beta_2 I_0^*}{I_1^*}\right) + \zeta_3(N - N^*)\left(\frac{\rho S}{N} - \frac{\rho S^*}{N^*}\right) + \zeta_4(P - P^*)\left(\frac{\omega_1 N}{P} + \frac{\omega_2 I_0}{P} + \frac{\omega_3 I_1}{P} - \frac{\omega_1 N^*}{P^*} - \frac{\omega_2 I_0^*}{P^*} - \frac{\omega_3 I_1^*}{P^*})\right) \quad (22)$$

$$\frac{dV}{dt} = (S - S^*)\left(\frac{\Lambda}{S} - \frac{\Lambda}{S^*}(\pi_1 + \pi_2)\beta_1(I_0 - I_0^*)\right) + \zeta_1(I_0 - I_0^*)((\pi_1 + \pi_2)\beta_1(S - S^*)) + \zeta_2(I_1 - I_1^*)(\pi_1 + \pi_2)\beta_2\left(\frac{I_0}{I_1} - \frac{I_0^*}{I_1^*}\right) + \zeta_3(N - N^*)\rho\left(\frac{S}{N} - \frac{S^*}{N^*}\right) + \zeta_4(P - P^*)\left(\omega_1 \frac{N}{P} - \frac{N^*}{P^*}\right) + \omega_2\left(\frac{I_0}{P} - \frac{I_0^*}{P^*}\right) + \omega_3\left(\frac{I_1}{P} - \frac{I_1^*}{P^*})\right) \quad (23)$$

$$\frac{dV}{dt} = (S - S^*)\left(\Lambda \frac{S^* - S}{SS^*}\right) - (\pi_1 + \pi_2)\beta_2(I_0 - I_0^*)(S - S^*) + \zeta_1((\pi_1 + \pi_2)\beta_1(S - S^*)(I_0 - I_0^*)) + \zeta_2(\pi_1 + \pi + 2)\beta_2(I_1 - I_1^*)\left(\frac{I_0 I_1^* - I_0^* I_1}{I_1 I_1^*}\right) + \zeta_3(N - N^*)\rho \frac{SN^* - S^*N}{NN^*} + \zeta_4(P - P^*)\left(\omega_1 \frac{NP^* - N^*P}{PP^*} + \omega_2 \frac{I_0 P^* - I_0^* P}{PP^*} + \omega_3 \frac{I_1 P^* - I_1^* P}{PP^*}\right) \quad (24)$$

$$\frac{dV}{dt} = \frac{-\Lambda(S - S^*)^2}{SS^*} - (\pi_1 + \pi_2)\beta_2(I_0 S - I_0 S^* - I_0 S + I_0^* S^*) + \zeta_1(\pi_1 + \pi_2)\beta_1(SI_0 - SI_0^* - S^* I_0 + S^* I_0^*) + \zeta_2 \frac{(\pi_1 + \pi_2)\beta_2}{I_1 I_1^*}(I_1 I_0 I_1^* - I_1^2 I_0^* - I_0(I_1^*)^2 + I_1 I_0^* I_1^*) + \zeta_3 \frac{\rho}{NN^*}(NSN^* - S^*N^2 - S(N^*)^2 + S^*N^*N) + \zeta_{41}\left(\frac{\omega_1}{PP^*}(NP^*P - N^*P^2 - N(P^*)^2 + N^*P^*P)\right) + \zeta_{42}\left(\frac{\omega_2}{PP^*}(PP^*I_0 - I_0^*P^2 - I_0(P^*)^2 + I_0^*P^*P)\right) + \zeta_{43}\left(\frac{\omega_3}{PP^*}(PP^*I_1 - I_1^*P^2 - I_1(P^*)^2 + I_1^*P^*P)\right) \quad (25)$$

$$\frac{dV}{dt} = \frac{-\Lambda(S - S^*)^2}{SS^*} - (\pi_1 + \pi_2)\beta_2 I_0(S - S^*) - (\pi_1 + \pi_2)\beta_2(I_0 S - S^* I_0^*) + \zeta_1(\pi_1 + \pi_2)\beta_1 I_0(S - S^*) - \zeta_1(\pi_1 + \pi_2)\beta_1 I_0^*(S - S^*) + \zeta_2 \frac{(\pi_1 + \pi_2)\beta_2}{I_1 I_1^*}(I_0 I_1^*(I_1 - I_1^*) - I_1 I_0^*(I_1 - I_1^*)) + \zeta_3 \frac{\rho}{NN^*}(-NS(N^* - N) + SN^*(N - N^*)) + \zeta_{41}\frac{\omega_1}{PP^*}(NP^*(P - P^*) - N^*P(P^* - P)) + \zeta_{42}\frac{\omega_2}{PP^*}(I_0P^*(P - P^*) - I_0^*P(P - P^*)) + \zeta_{43}\frac{\omega_3}{PP^*}(I_1P^*(P^* - P) - I_1^*P(P - P^*)) \quad (26)$$

Now replacing

$$\zeta_1 = \frac{1}{(\pi_1+\pi_2)\beta_1}$$
$$\zeta_2 = \frac{1}{(\pi_1+\pi_2)\beta_2}$$
$$\zeta_3 = \frac{1}{\rho}$$
$$\zeta_{41} = \frac{1}{\omega_1}$$
$$\zeta_{42} = \frac{1}{\omega_2}$$
$$\zeta_{43} = \frac{1}{\omega_3} \tag{27}$$

The following is obtained:

$$\begin{aligned}\frac{dV}{dt} < 0 = &\frac{-\Lambda(S-S^*)^2}{SS^*} - (\pi_1+\pi_2)\beta_2 I_0(S-S^*) - (\pi_1+\pi_2)\beta_2(I_0S - S^*I_0^*) + I_0(S-S^*) - I_0^*(S-S^*)\\ &+\frac{1}{I_1I_1^*}(I_0I_1^*(I_1-I_1^*) - I_1I_0^*(I_1-I_1^*)) + \frac{1}{NN^*}(-NS(N^*-N)+SN^*(N-N^*))\\ &+\frac{1}{PP^*}(NP^*(P-P^*) - N^*P(P^*-P)) + \frac{1}{PP^*}(I_0P^*(P-P^*) - I_0^*P(P-P^*))\\ &+\frac{1}{PP^*}(I_1P^*(P^*-P) - I_1^*P(P-P^*))\end{aligned}$$
$$\frac{dV}{dt} < 0 \tag{28}$$

Hence, system (1) is globally stable at endemic equilibrium.

3.5 Bifurcation

Bifurcation analysis helps to recognize the changes in the solution. Here, the center manifold theory is used, to identify the change by considering the coefficient of system (1)[4]. If the coefficients a and b are positive then it becomes backward bifurcation, and if a is positive, b is nonpositive, then it is forward bifurcation or transcritical bifurcation.

The bifurcation behavior of system (1) is explained by the following theorem:

Theorem 3.4. If a is positive and b is nonpositive, then the system exhibits transcritical bifurcation.

Proof. When $R_0 = 1$, the bifurcation parameter β_1 is,

$$\beta_1^* = \frac{((\pi_1+\pi_2)\beta_2 + \gamma_2 + \omega_2)(\phi + \rho - (\gamma_1+\gamma_3+\alpha))}{(\pi_1+\pi_2)\Lambda} \tag{29}$$

From the Center manifold, consider S, I_0, I_1 as x_1, x_2 and x_3 respectively. Then the equations become

$$\frac{dx_1}{dt} = \Lambda + (\gamma_1+\gamma_3+\alpha-(\phi+\rho))x_1 - (\pi_3+\pi_2)\beta_1^* x_1x_2$$
$$\frac{dx_2}{dt} = (\pi_1+\pi_2)\beta_1^* x_1x_2 - (\pi_1+\pi_2)\beta_2 x_2 - (\gamma_2+\omega_2)x_2$$
$$\frac{dx_3}{dt} = (\pi_1+\pi_2)\beta_2 x_2 - \gamma_2 x_3 - \omega_3 x_3 \tag{30}$$

The disease-free equilibrium points for $x_1, x_2,$ *and* x_3 are $\frac{\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha)}, 0$ and 0 respectively. Now the

The Jacobian matrix of (30) is

$$J = \begin{pmatrix} \gamma_1+\gamma_3+\alpha-(\phi+\rho)-(\pi_1+\pi_2)\beta_1^* x_2 & -(\pi_1+\pi_2)\beta_1^* x_1 & 0 & 0 \\ (\pi_1+\pi_2)\beta_1^* x_2 & (\pi_1+\pi_2)\beta_1^* x_1-(\pi_1+\pi_2)\beta_2+\gamma_2+\omega_2 & 0 & 0 \\ 0 & (\pi_1+\pi_2)\beta_2 & -(\gamma_2+\omega_3) & 0 \\ \rho & 0 & 0 & -\omega_1 \end{pmatrix} \quad (31)$$

Substituting the values of x_1, x_2 and x_3 in (31), we obtain

$$J = \begin{pmatrix} \gamma_1+\gamma_3+\alpha-(\phi+\rho) & \frac{-(\pi_1+\pi_2)\beta_1^*\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha)} & 0 & 0 \\ 0 & \frac{-(\pi_1+\pi_2)\beta_1^*\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha)}-(\pi_1+\pi_2)\beta_2+\gamma_2+\omega_2 & 0 & 0 \\ 0 & (\pi_1+\pi_2)\beta_2 & -(\gamma_2+\omega_3) & 0 \\ \rho & 0 & 0 & -\omega_1 \end{pmatrix} \quad (32)$$

To determine the right eigenvector, u=(u_1, u_2,u_3)^T) and $Ju = 0$,

$$(\gamma_1+\gamma_3+\alpha-(\phi+\rho))u_1 - \frac{(\pi_1+\pi_2)\beta_1^*\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha(\phi+\rho))}u_2 = 0$$

$$\left(\frac{(\pi_1+\pi_2)\beta_1^*\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha)}u_2 - ((\pi_1+\pi_2)\beta_2+\gamma_2+\omega_2)\right)u_2 = 0$$

$$(\pi_1+\pi_2)\beta_2 u_2 - (\gamma_2+\omega_3)u_3 = 0 \quad (33)$$

From (33), the values are

$$u_1 = \frac{-(\pi_1+\pi_2)\beta_1^*\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha-(\phi+\rho))^2}; u_2 = 1 \text{and } u_3 = \frac{\gamma_2+\omega_3}{(\pi_1+\pi_2)\beta_2} \quad (34)$$

To compute the left eigenvectors, $v = (v_1, v_2, v_3)$ and $vJ = 0$,

$$(\gamma_1+\gamma_3+\alpha-(\phi+\rho))v_1 = 0$$

$$-\left(\frac{(\pi_1+\pi_2)\beta_1^*\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha(\phi+\rho))}\right)v_1 - ((\pi_1+\pi_2)\beta_2+\gamma_2+\omega_2)v_2 + (\pi_1+\pi_2)\beta_2 v_3 = 0 \quad (35)$$

$$-(\gamma_2+\omega_3)v_3 = 0$$

From(35), the values are

$v_1 = v_3 = 0; v_2 = 1$ (36)

Thus the value of v_2 should be calculated with the condition v.u=1. The function f_2 and its partial derivatives are considered. The only nonzero partial derivatives are given below:

$$f_2 = (\pi_1+\pi_2)\beta_1^* x_1 x_2 - (\pi_1+\pi_2)\beta_2 x_2 - (\gamma_2+\omega_2)x_2$$

$$\frac{\partial^2 f_2}{\partial x_2 \partial x_1} = (\pi_1+\pi_2)\beta_1^* = \frac{\partial^2 f_2}{\partial x_1 \partial x_2}$$

$$\frac{\partial^2 f_2}{\partial x_2 \partial \beta_1^*} = (\pi_1+\pi_2)x_1 = \frac{(\pi_1+\pi_2)\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha)} \quad (37)$$

Now to find the coefficient signs ‘a’ and ‘b’:

$$a = v_2 u_1 u_2 \frac{\partial^2 f_2}{\partial x_2 \partial x_1}$$
$$= (1)\frac{-(\pi_1+\pi_2)\beta_1^*\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha-(\phi+\rho))^2}(\pi_1+\pi_2)\beta_1^*$$
$$= \frac{-(\pi_1+\pi_2)^2(\beta_1^*)^2\Lambda}{(\phi+\rho)-(\gamma_1+\gamma_3+\alpha-(\phi+\rho))^2} \quad (38)$$

$a < 0$

$$b = v_2 u_2 \frac{\partial^2 f_2}{\partial x_2 \partial \beta_1^*}$$
$$= \frac{(\pi_1+\pi_2)\Lambda}{(\phi+\rho)-(\gamma_1)+\gamma_3+\alpha}$$
$$= \frac{R_0((\pi_1+\pi_2)\beta_2+\gamma_2+\omega_2)}{\beta_1^*} \quad (39)$$

$b > 0$

Therefore, the system exhibits transcritical bifurcation. i.e., forward bifurcation.

4 NUMERICAL SIMULATION

The parameter values are calculated and presented in Table (3). Here, T(*total number of plants*) = 1000, S = 900, I_0 = 300, I_1 = 500, N = 900 and P = 500. The values are assigned to the parameters by the survey conducted in the Tiruvannamalai area. In **Figure (2)**, the graph represents all state variables with cultural controls. When $R_0 < 1$, the initial and severe infection decreases whereas the other populations increase. Figure (3) depicts the graph of the susceptible population with four cases: 1. Without cultural controls, 2. With cultural controls, 3. If γ_1 and γ_2 are applied and 4. Only γ_2 is applied. This graph shows that using all the culture controls in the susceptible state increases the plant population.

Table 2: Values of the parameters

Parameters	Values	Parameters	Values
Λ	0.9	α	0.05
π_1	0.5	π_2	0.5
φ	0.01	ρ	0.7
β_1	0.5	β_2	0.8
γ_1	0 or 0.5	γ_2	0 or 1
γ_3	0 or 0.5	ω_1	0.9
ω_2	0.3	ω_3	0.5
h	0.8		

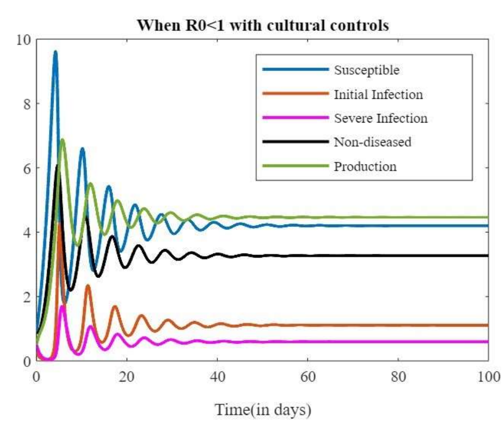

Figure 2: When $R0 < 1$ with cultural controls

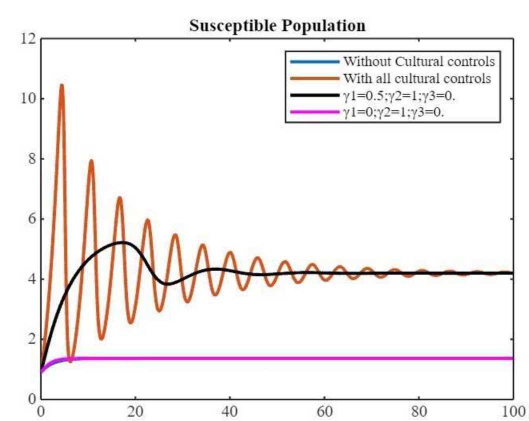

Figure 3: Susceptible Population

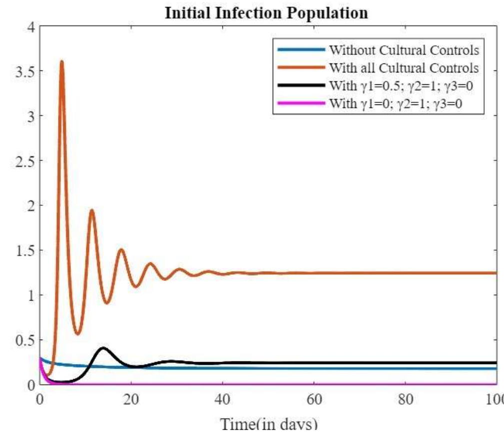

Figure 4: Initial Infection Population

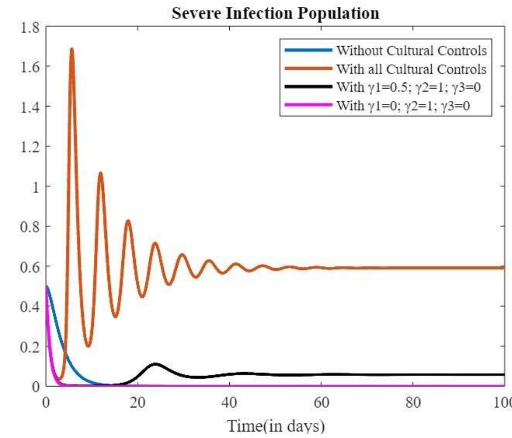

Figure 5: Severe Infection Population

Figure (4) and Figure (5) show the graphs of the initial and severe infection populations from various perspectives. The four cases in the graphs are as follows: 1. Without cultural controls 2. With all the cultural controls, 3. When only two cultural controls γ_1 and γ_2 and 4, are used. If γ_3 is applied, only. Both graphs indicate that if the infection is present in the plant population, the use of γ_2 alone decreases the initial and severe infection in the plant population. The other two cultural controls increased the population of infected plants.

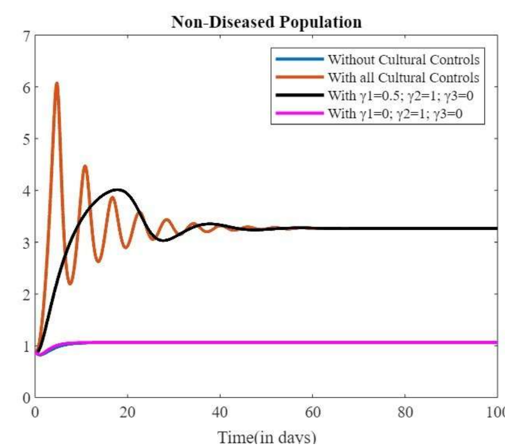

Figure 6: Nondiseased population without controls for different values of γ_1 and γ_2

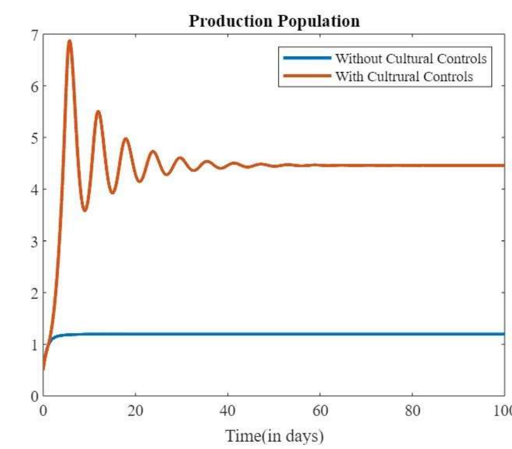

Figure 7: Production population without and with controls γ_1 and γ_2

The graph of the nondiseased cases is displayed in **Figure (6)**, which includes the same four cases as **Figures (4) and (5)**. The non-diseased population increases in two cases: one in which all cultural controls are used and the other in which only two cultural controls, γ_1 and γ_2, are used. **Figure (7)** presents the graph of the production population, which shows that using all cultural control increases the production in the plant population.

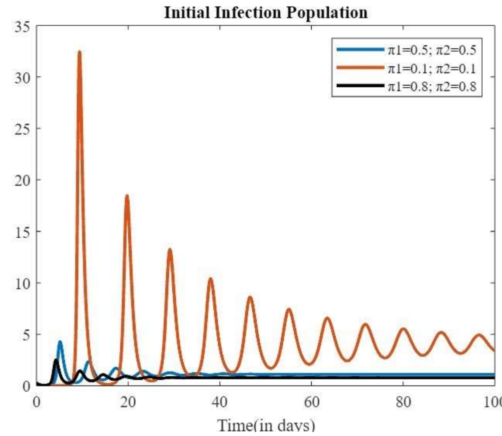

Figure 8: Initial infection population for Different values of π_1 and π_2

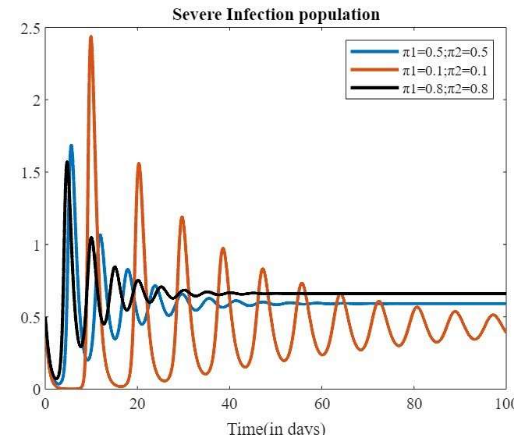

Figure 9: Severe infection population for Different values of π_1 and π_2

Figure (8) and Figure (9) show the graphs of initial and severe infection with the causes of disease π_1 and π_2. In the two scenarios, $\pi_1 = \pi_2 = 0.5$ and $\pi_1 = \pi_2 = 0.8$, and the initial and severe infections are stable.

In the alternative scenario, $\pi_1 = \pi_2 = 0.1$ reveals fluctuations in the initial and severe infections.

5 CONCLUSION

In this study, the two main causes of banana wilt disease were developed via a mathematical model along with cultural controls. From the two equilibrium points, if $(\varphi+\rho) > \gamma_1 + \gamma_3 + \alpha$, the illness is not prevalent in the plant population if the percentage of noninfected plants (ρ) is higher than the percentage of suckers (α); if not, the disease will be endemic to the plant population. The reproduction number is determined by the next-generation matrix. The reproduction number is used to examine the local stability of both equilibria. The global stability was examined through the Lyapunov function. Bifurcation is shown to be transcritical. Numerical simulation is performed to validate theoretical analysis. Thus, this study aims to demonstrate the effects of applying cultural controls which significantly help susceptible, nondiseased and productive populations without infection. Specifically, if the cultural controls γ_1 (proper drainage) and γ_2 (removal of the infected plants) are used in the early stages, the number of infected plants decreases. As a result, γ_3 (intercropping)in the initial and severe infection states increases the number of infected individuals. The diseased population may be stable if the two main causes, π_1 (humidity) and π_2 (temperature and climatic change), are low and balanced. If not, the population will be unstable.

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