



**International Journal of Biology, Pharmacy  
and Allied Sciences (IJBPAS)**

*'A Bridge Between Laboratory and Reader'*

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## MATHEMATICAL MODELLING OF SECOND WAVE OF COVID 19 IN DIFFERENT STATES IN INDIA

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Received 8<sup>th</sup> April 2022; Revised 3<sup>rd</sup> May 2022; Accepted 25<sup>th</sup> July 2022; Available online 1<sup>st</sup> Feb. 2023

<https://doi.org/10.31032/IJBPAS/2023/12.2.6846>

### ABSTRACT

**Background:** India was under a grave threat from the second wave of the COVID-19 pandemic, particularly at the beginning of March 2021. The situation became critical as the cases increased alarmingly around the months April to August 2021 compared to the first wave of coronavirus. The Indian government has implemented various control measures such as lockdowns putting travel restrictions at various stages to restrict the spread of the virus from the initial outbreak of the pandemic. Recently, we have studied the susceptible-exposed-Quarantined-infectious-removed (SEQIR) dynamic modeling of the epidemic evolution of COVID-19 in India with the help of appropriate parameters in this present article, we extend our analysis to estimate and analyze the number of infected individuals during the second wave of COVID-19 in India with the help of the above SEQIR model. Methods: We integrated the most updated COVID-19 epidemiological data into the Susceptible-Exposed-Quarantine-Infectious-Removed (SEQIR) model to derive the epidemic curve. We implemented parameter estimations to predict the nature of the epidemic.

**Results:** Our findings show that the people's effort along with governmental actions such as execution of restrictions is an important factor to control the pandemic in the present situation and the future.

**Conclusions:** Our dynamic SEQIR model was effective in predicting the COVID-19 epidemic peaks and sizes.

**Keywords:** Coronavirus disease 2019 (COVID-19; modeling; Susceptible-Exposed-Quarantined-Infectious-Removed (SEQIR)

## INTRODUCTION

India has been through the consequences of the coronavirus (COVID) since 2019. COVID 19 has affected our country by spreading rapidly and is believed to be the worst pandemic after the influenza outbreak in, 1918 which claimed millions of

lives. Compared to the first wave, the second wave has created more devastation. The central government of India has adopted various measures in an attempt to mitigate the effects of the pandemic.

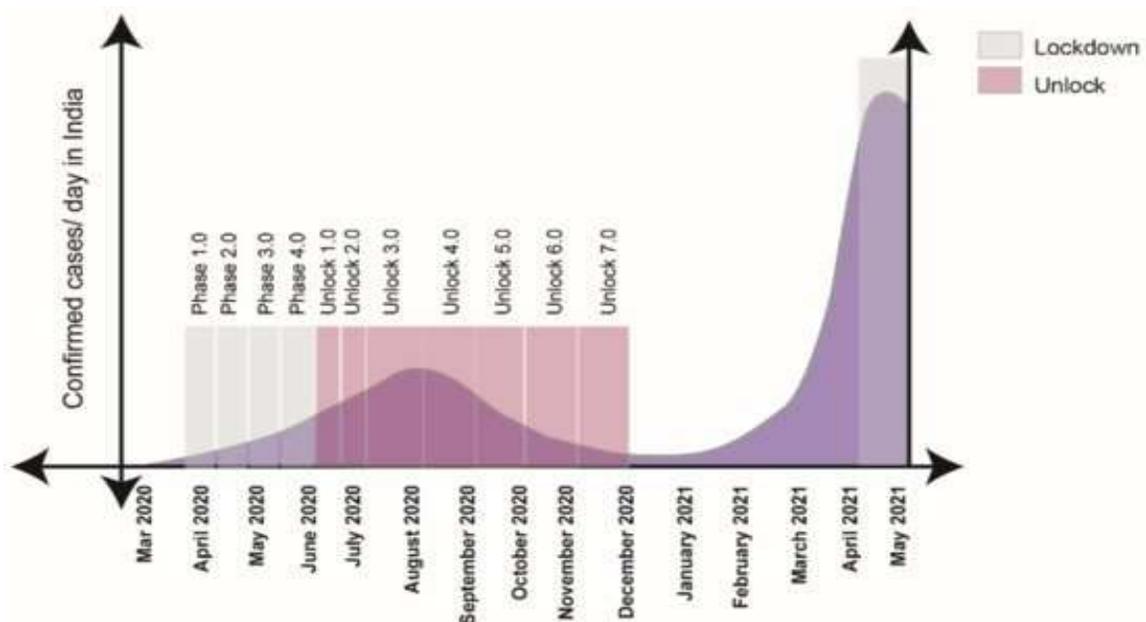


Figure 1: Illustration representing the first wave, the nationwide lockdown in India started at the end of March and lasted up to June 2020, followed by unlocking from the end of June 2020 to December 2020. In the second wave, the state-wise lockdown has been implemented from April 2021 to the present [1]

However, these measures could not control the extension of the virus completely because of various factors like the mutation of the virus, seasonal immunity levels, and reinfections leading to an upsurge in the

number of infections [2]. In March 2021, the first case of the second wave of COVID 19 has been reported in India by the ministry of health, and by April, maximum cases have been reported in India (Figure 1) with the

majorly affected states being Maharashtra, Tamilnadu, Kerala, Karnataka, Telangana, Andhra Pradesh, Delhi, Uttar Pradesh, West Bengal, Odisha, Rajasthan, Chhattisgarh,

Haryana, Bihar, Gujarat, Madhya Pradesh, Assam, Punjab, Jharkhand, Uttarakhand [3] (Shown in Figure 2),

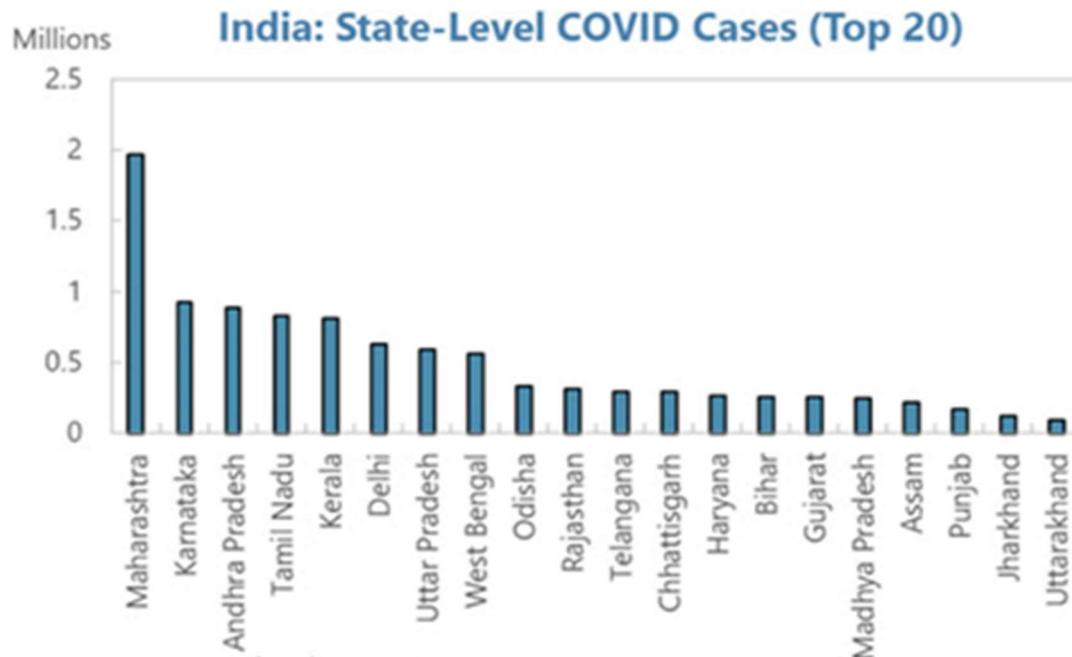


Figure 2: (A graph showing the twenty worst-affected states with the number of cases in April due to the second wave of COVID 19)

To understand the spread of this disease, several studies have been conducted and various mathematical models have been developed by incorporating Mathematics and Statistics providing a clear insight into the pandemic [4-7]. The probability and severity of several diseases like foot and mouth disease, SARS, Ebola, and Zika [8] are predicted by mathematical modeling and simulations. In the same way, Mathematical models are used to simulate the spread of Coronavirus [9-11]. Sun *et al.*, 2020 proposed a mathematical model for COVID 19

infection based on the data analysis present in Wuhan, with identification that the measures taken by the government reduced the number of infections. A flexible model is projected to calculate the growth of COVID 19 in a certain period for the USA, Slovenia, Iran, and Germany [12]. The correlation between the size of the cities and the spread of COVID 19 in Brazil is studied by Ribeiro, studied the transmission and mortality rates during certain periods using the SEIR model, and these simulations are well applied after the quarantine period in Spain and Germany

for the post-COVID infection records [13, 14]. A two-way statistical model is proposed, in which the future scenario is calculated based on the occurrence of the first one [15]. These analyses are considered to have many hitches and, in this context, Chanu and Singh [16], developed a study by using the stochastic SEQIR model to analyze the epidemic in India. Similarly, a discrete-time stochastic SEIHR model was proposed [17]. A time delay of the stochastic SEIHR model is proposed [18] and the stochastic SEIR model is developed by Faranda and Alberti [19], to analyze the impact of relaxation measures implemented in France and Italy.

Here, we collected the COVID 19 data from the authoritative sources: The National Health Commission of India until the article is completed (September 2021). Then we carried out routine dynamical modeling on the epidemic of COVID-19 in the five most interesting regions in India (Maharashtra, Delhi, Tamilnadu, Kerala, and West Bengal) [20]. This design aims to minimize the frequency of the errors due to their extremely large infected populations compared to other regions. Considering all the studies conducted so far hitherto, we propose to analyze the second wave COVID

19 infection data of ten majorly infected states using the SEQIR (Susceptible, Exposed, Quarantined, Infected, and Recovered) model with standard incidence, together with deterministic and stochastic models.

## MODEL FORMULATION

To depict the widespread of the second wave of COVID-19 which outbreaked in India, we streamlined the SEQIR model for the five majorly affected states, i.e.  $\{S(t), E(t), Q(t), I(t), R(t)\}$  denoting at time  $t$  the respective number of the *susceptible cases, exposed cases, infectious cases, quarantined cases and recovered cases*. The adding of newly quarantined data replaces the classical SEIR model [21]. Their relations are given in **Figure 3** and characterized by a group of ordinary differential equations. Constant  $N(t) = S(t) + E(t) + Q(t) + I(t) + R(t)$  is the total population in a certain region. The coefficients  $\{\Lambda, \alpha, \beta, \gamma, \delta, \lambda, k(t)\}$  represent the rate of inflow in the Indian population due to new child-births or immigration to the country; transmission rate, infection rate, average latent time, average quarantine time, and cure rate, separately.  $N$  represents the total population at any instant of the time.

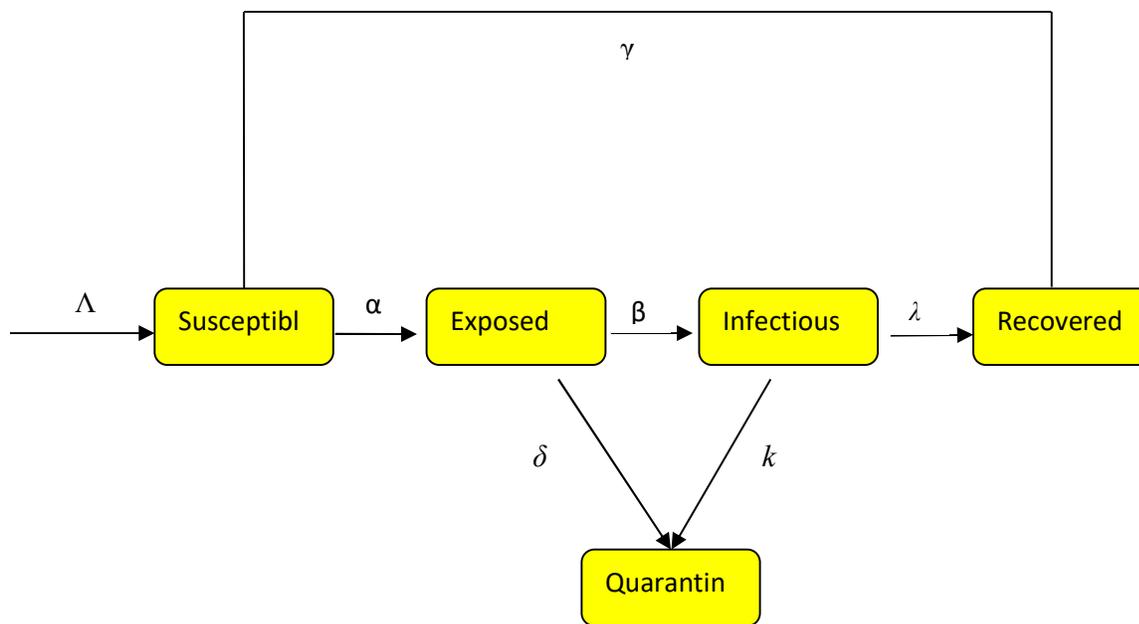


Figure 3: Flow chart of, the SEQIR model for the five majorly affected states

Through the basis of mentioned assumptions, the transmission of COVID 19 is governed by the following differential equations :

$$\begin{aligned} \frac{dS(t)}{dt} &= -\beta \frac{S(t)I(t)}{N} - \alpha S(t) \\ \frac{dE(t)}{dt} &= \beta \frac{S(t)I(t)}{N} - \gamma E(t) \\ \frac{dQ(t)}{dt} &= \delta I(t) - \lambda(t)Q(t) - k(t)Q(t) \\ \frac{dI(t)}{dt} &= \gamma E(t) - \delta I(t) \\ \frac{dR(t)}{dt} &= \lambda(t)Q(t) \end{aligned} \dots \text{(Equation 1)}$$

**Numerical Analysis**

We solve the differential Equations (1) by using a Euler finite-difference scheme similar to work carried out by Carcione [22]

where the time variable as  $t = n dt$ , where  $n$  is a natural number and  $dt$  is the time step. The equations become:

$$\begin{aligned} S^{n+1} &= S^n + dt (\Lambda - \alpha S^n - \beta S^n I^n / N^n), \\ E^{n+1} &= E^n + dt [\beta S^n I^n / N^n - (\alpha + \delta) E^n], \\ Q^n &= - (S^n + E^n + I^n + R^n) (t), \\ I^{n+1} &= I^n + dt [\delta E^n - (\gamma + \delta + \alpha) I^n], \\ R^{n+1} &= R^n + dt (\gamma I^n - \mu R^n), \end{aligned}$$

Where  $Q^n$  is the number of quarantined people  $n$ . This algorithm yields to an equilibrium, i.e.,  $S_\infty + E_\infty + Q_\infty + I_\infty + R_\infty = N_0$  for  $t \rightarrow \infty$ . (Equation 2)

Where, the initial conditions are noted to be  $S_{(0)} \geq 0, E_{(0)} \geq 0, Q_{(0)} \geq 0, I_{(0)} \geq 0, R_{(0)} \geq 0$ .

**Reproduction Ratio**

It is noted that here we assume the cure rate  $\lambda$  is time-dependent. As confirmed in **Figure 3**, the cure rate  $\lambda(t)$  is gradually increasing with time, and to foresee a possible spread of an infectious disease in a population, the basic reproduction number ( $R_0$ ) is an epidemiological metric. This represents the average number of secondary cases resulting from a single case in a population susceptible to the disease. If an  $R_0 > 1$ , the disease spreads rapidly with an increase in the number of novel cases. The basic reproduction number for the SEQIR model is represented by

$$R_0 = \beta \alpha + \gamma.$$

### Estimation of the Parameters

According to the daily official reports of India, the cumulative numbers of infected, quarantined cases, and recovered cases are available in public and since the latter two are directly related to the first one through the time-dependent recovery rate the numbers of quarantined cases  $Q(t)$  play a key role in our modeling. Such an optimization problem could be solved automatically by using the simulating

algorithm. A major difficulty is how to overcome the time problem. To this end, we firstly prefix the latent time  $\gamma^{-1}$ , which is normally estimated within a specific time. And then for each static  $\gamma^{-1}$ , we explore its influence on other parameters, initial values, as well as the population dynamics of quarantined cases and infected cases which is similar to the work carried out by, Liangrong [23]. From Figure 4, to produce the same outcome, the protection rate  $\alpha$  and the reciprocal of the quarantine time  $\delta^{-1}$  are both decreasing with the latent time  $\gamma^{-1}$ , which is consistent with the fact that longer latent time requires longer quarantine time. The initial values of exposed cases and infectious cases increase with the suppressed time. This agrees with the observation, that COVID-19 has an extremely strong infectious ability. Nearly every exposed person will be infected after direct contact with the COVID-19 patients. And based on a reasonable estimation of the total number of infected cases, the latent time is finally determined as 3-4 days.

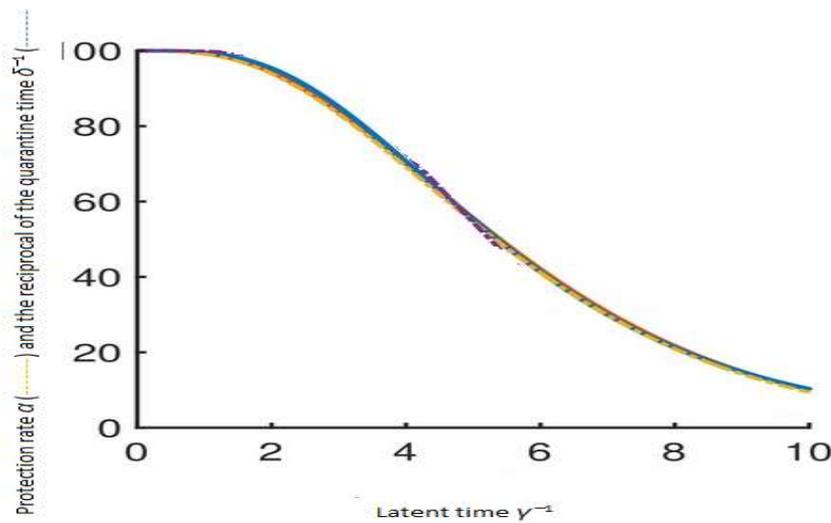


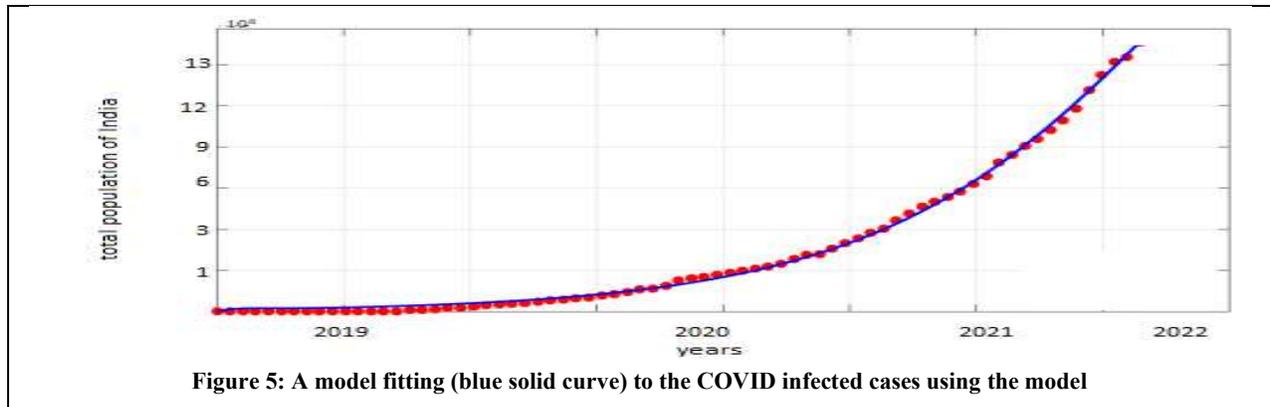
Figure 4: A graph representing the protection rate  $\alpha$  and the reciprocal of the quarantine time  $\delta^{-1}$  vs the latent time  $\gamma^{-1}$

Investigation was carried out on the curve fitting to the actual confirmed COVID-19 cases from 2020 March, to 2022 until the end of second wave in India, using the estimated parameters of COVID-19 model from the existing literature cited in the **Table**

1. In this case from the present resources the average inflow of population ( $\Lambda$ ) is found to be, 67,446.82 and from the sources the calculated and fitted values for the parameters were taken and a model fitting curve is established in the **Figure 5**.

Table 2: Estimated and fitted values for the COVID 19 parameters [24-28]

Parameters	Description	Baseline value	Source
$\Lambda$	rate of inflow in the Indian population due to new child-births or immigration to the country	The birth rate is 18.2/1000 per year So the daily recruitment rate in India is 67, 446.82.	Mohak <i>et al.</i> , (2021)
$\alpha$	Transmission rate	0.9	Buitrago-Garcia <i>et al.</i> , (2020), Mohak <i>et al.</i> , (2021)
$\beta$	Infection rate	4.3	li <i>et al.</i> , (2020)
$\gamma$	Average latent time	3.49	He <i>et al.</i> , (2020), Li <i>et al.</i> , (2020), Liu <i>et al.</i> , (2020)
$\delta$	Average quarantine time	0.1315	He <i>et al.</i> , (2020), Ali <i>et al.</i> , (2020)
$\lambda$	Cure rate	1.6594	Buitrago-Garcia <i>et al.</i> , (2020), Mohak <i>et al.</i> , (2021)
N	The total population at any instant of the time	1,352,642,280	Buitrago-Garcia <i>et al.</i> , (2020), Mohak <i>et al.</i> , (2021)



## RESULTS

### Interpretation of the public data for the second wave of coronavirus

We applied the SEQIR model to infer the public data on the cumulative numbers of quarantined cases, recovered cases, and closed cases from March 2021 to August 2021 which are published daily by the health commission of India. Our preliminary study includes five states, *i.e.* Maharashtra [29], Delhi [30], Tamilnadu [31], Kerala [32], and West Bengal [33]. Through all the encompassing models, the top values for unknown model parameters and initial conditions, the observed cumulative numbers of infected cases, active cases, and recovered cases are determined and abridged in **Table 1**. Several remarkable facts could be immediately learned from **Table 2** and **Figure 6**. Firstly, the protection rate of Maharashtra is significantly lower than other regions (**Figure 6**), the estimated number of total infected cases in five regions are all

significantly larger, suggesting the COVID-19 has already spread out nationwide at that moment.

### B. Estimation of the Wide Spread of Corona Virus

We evaluated various methods of the SEQIR model of the standard initial values to study the progression of the COVID 19 infected population of India and above mentioned five Indian states concerning the parameters (**Table 3, Figure 7**).

### C. Inverse inference on the epidemic of COVID-19

Besides the prediction, the early curve of the COVID-19 outbreak is also critical for our understanding on its epidemic as well as future prevention. To this end, we carried out inverse interpretation (**Figure 8**) to explore the dynamics of COVID-19 since it started during the second wave of coronavirus. The number of total infected cases follows a nice exponential curve with the doubling time around-6 days.

Table 2: The confirmed, actively infectious, and recovered cases of the five states as of April 2021

PARAMETERS	MAHARASHTRA	DELHI	TAMILNADU	KERALA	WEST BENGAL
CONFIRMED	6,602,961	5,939,239	2,695,216	4,915,331	3,044,132
INFECTIOUS	1,000,506	84000	13,034	77,964	56,788
RECOVERED	6,435,439	5,568,456	2,646,163	4,808,775	3,003002

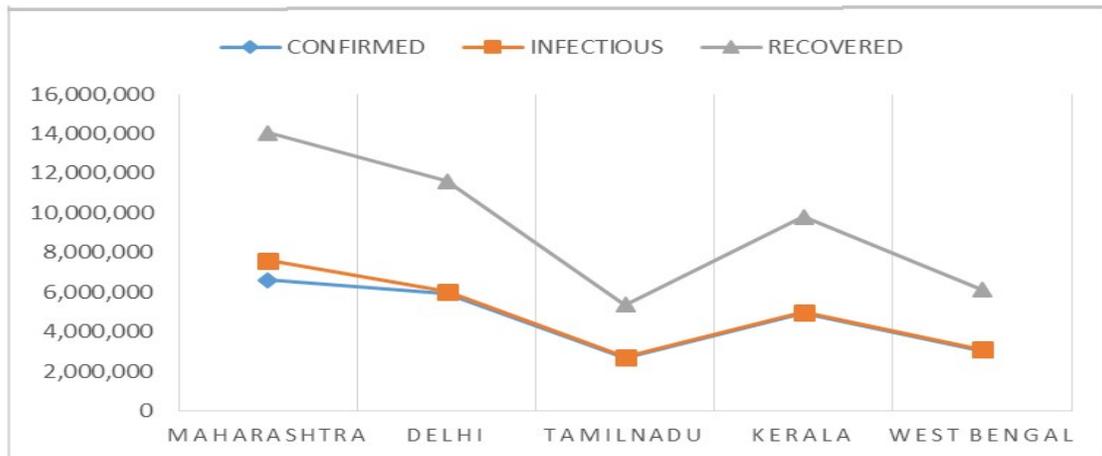


Figure 6: Representing the graphical format for the confirmed, infectious, and recovered cases of five states of India’s respective

Table 7: Initial Values of parameters in the states of Maharashtra, Delhi, Tamil Nadu, Kerala, and West Bengal (source government websites, and calculated)

PARAMETERS	MAHARASHTRA	DELHI	TAMILNADU	KERALA	WEST BENGAL
$S_{(0)}$	3,301,480	2,390,300	1,390,300	1,515,300	1,390,300
$E_{(0)}$	73,500	80,150	4700	35,000	12,000
$Q_{(0)}$	78000	67054	4500	32000	12000
$I_{(0)}$	70,016	9000	1450	2500	3000
$R_{(0)}$	3,234,400	2,408,700	1,260,300	1,430,300	1,140,300

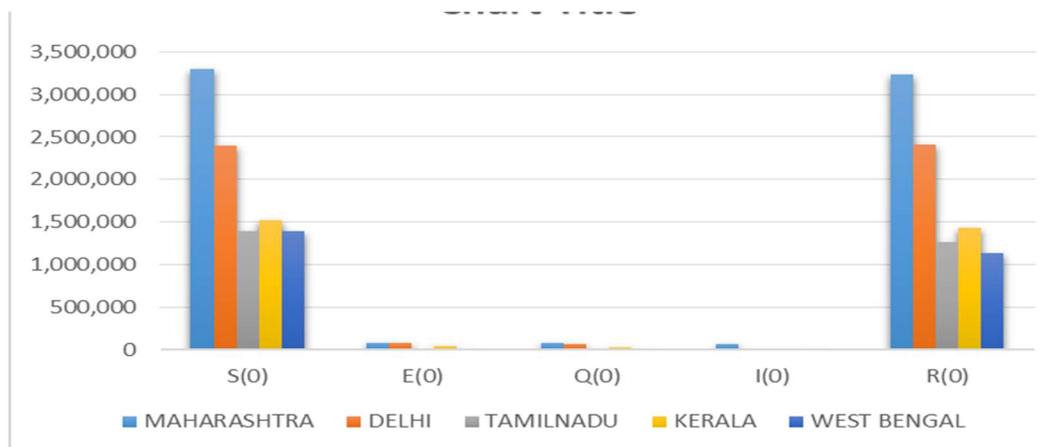
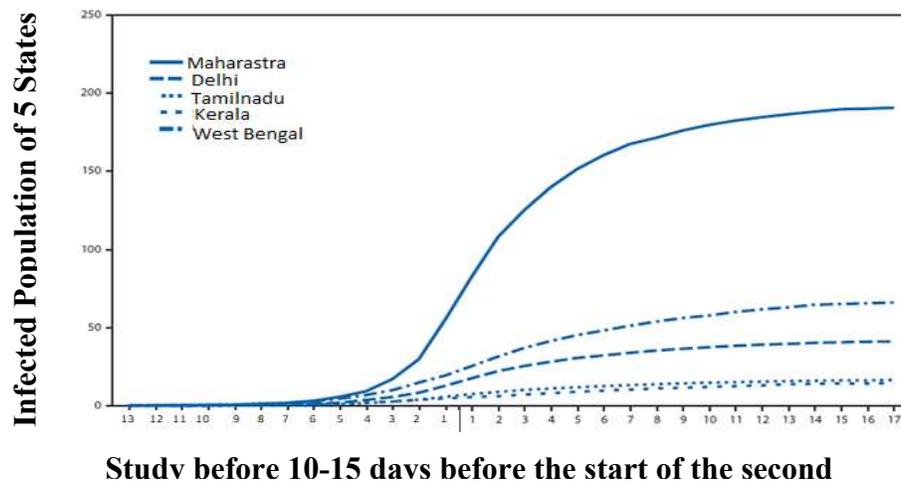


Figure 7: Graphical representation of the SEQIR parameters of the 5 states of India



**Figure 8: Representing the Inverse inference on the total population of the 5 states of India before cases start**

## DISCUSSION AND CONCLUSION

We proposed the SEQIR model approach for modeling and predicting the second wave COVID-19 epidemic in India. We simulated the second wave using the modified SEQIR method which is a modification of the SEIR method, where we incorporated Quarantine data to study the effect of the latent time on the spread of the Coronavirus second wave. Our method appropriately assimilates the effect of exposed and infectious cases, which is difficult for traditional statistics analysis. A new quarantined state, replace the original  $R$  state in the classical SEIR model and correctly accounts for the daily reported confirmed infected cases and recovered cases. This method can be adopted by other countries also.

In accordance with the acquired data, India was recorded to be the second-highest COVID-19 knocked country. Maharashtra, Delhi, Tamil Nadu, Kerala, and West Bengal evidently reported high cases in India. The evolution of the total number of positive cases from March 2020, to May 2021, with respect to time, is graphically displayed in **Figure 1** and as per the observations and available data, the second wave of corona spread started during the first week of March 2021 in India and its provinces. This work is in agreement with the work carried out by Ranjan *et al.*, (2020). In this study, we have selected the data set of positive cases of COVID-19 in most five states of India, namely Maharashtra, Delhi, Tamil Nadu, Kerala, and West Bengal. The data set has been retrieved for the second wave. We

performed a numerical simulation of our proposed model. (Equations 1 and 2) We first carried out the best fit model to graphically represent the number of cases present in India from the available data and then we simulated parameters for the five states where Maharashtra recorded the high number of cases. Similar works were carried out earlier by *Chanu and Singh, 2020* who simulated the data using the same SEQIR model and similar to our conclusions it is mentioned that policies like social distancing and quarantine have important roles in controlling the pandemic.

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