



**International Journal of Biology, Pharmacy
and Allied Sciences (IJBPAS)**

'A Bridge Between Laboratory and Reader'

www.ijbpas.com

**A STUDY OF cLFV DECAY $\mu \rightarrow e + \gamma$ FOR DIFFERENT VACUUM
ALIGNMENTS OF TRIPLET SCALAR FLAVON IN LINEAR SEESAW
MODEL**

DEVI MR* AND BORA K

Department of Physics, Gauhati University, Guwahati- 781014, Assam, India

*Corresponding Author: Dr. Maibam Ricky Devi: E Mail: deviricky@gmail.com

Received 10th June 2021; Revised 11th July 2021; Accepted 20th Aug. 2021; Available online 15th Jan. 2022

<https://doi.org/10.31032/IJBPAS/2022/11.1.1072>

ABSTRACT

We present a neutrino model realized by linear seesaw mechanism incorporating A_4 flavour symmetry. The light neutrino mass matrix is obtained from the model which is compared with that obtained experimentally. This gives us a set of six real equations which is later solved to obtain free parameters of neutrino mixing, i.e., lightest neutrino mass, Dirac CP violating phase δ and two Majorana phases, α and β . A full parameter scan of the recent 3σ global fit values of neutrino oscillation parameters is done to find the desired unknown parameters. Using this data of neutrino oscillation parameters we find the branching ratio of the cLFV decay $\mu \rightarrow e + \gamma$ for our LSS model, which is found to lie within present limits of the same. Since linear seesaw model is a low scale model, this is testable in experiments, as well as prediction of cLFV decay obtained in this model can be tested when future measurements are done.

INTRODUCTION

Matter around us is made up of fundamental particles like proton, neutrons, electrons (charged lepton) etc, and protons and neutrons are made up of more fundamental particles called quarks. The Standard Model (SM) of

particles has been a very successful theory in explaining many experimentally observed features, but cannot explain some of them, like - mass of neutrinos, dark matter (DM) etc. Neutrinos are very light, electrically neutral particles, whose mass has been established

experimentally to be very very low. Mass of electron is ~ 0.5 MeV, while that of neutrino is < 0.1 eV. One eV is the energy equivalent of work done in moving an electron through a potential difference of 1 Volt. Neutrinos are very very weakly interacting particles, and millions of them are passing through us all the time, which come to earth from Sun, other astrophysical, cosmological and extra galactic sources. Existence of neutrinos was established in nuclear beta decay experiments. On the other hand, dark matter is that matter which can be seen with the help of light, but that interacts with the usual visible matter through gravity. SM is a gauge theory, that can explain strong, electromagnetic and weak interactions (but not gravity) among fundamental particles. Quarks and leptons form SU(2) flavour doublet, while quarks form triplet of colour quantum number.

While mass of all other matter particles, like charged leptons and quarks can be explained with the help of Higgs mechanism, it cannot explain the tiny mass of neutrinos, as left handed neutrinos do not exist in the SM due to parity violation in weak interactions. And hence seesaw mechanism come to our rescue, that can explain tiny mass of neutrinos if a right handed neutrino is added to the theory. Also, canonical type-I and type-II seesaw mechanisms have a very high seesaw scale

than cannot be tested in manmade laboratory experiments, and hence low scale seesaw models, like the Linear seesaw (LSS) models become phenomenologically more interesting theories to explain the tiny mass of neutrinos. Also, discrete flavour symmetries are used to explain the observed flavour structure of fundamental particles, which should be broken by the vacuum expectation value (VEV) of the fields called flavons, as flavour symmetry is not observed at low energies ~ 100 GeV.

It has been observed experimentally that as neutrinos travel, they change flavour (type), and this is called flavour conversion or neutrino oscillation. This can be explained through neutrino mixing, if neutrinos have non-zero mass. Mixing occurs in the case of quarks too. This is a quantum mechanical phenomenon. If neutrinos are Majorana (self-conjugate) particles, then generation of neutrino mass term violates lepton number by 2 units. However, charged lepton flavour violation is a very rare process, which should not occur ideally. One of the key highlight in the construction of neutrino mass model is the study of cLFV decay processes such that the model is testable in ongoing and future colliders in order to find signature of lepton number violating particles and lepton flavour violating processes.

In this work, we propose a linear seesaw

model [1], [2] , with A_4 flavour symmetry and some other symmetries (necessary to avoid unwanted terms in the Lagrangian) to find the Branching ratio of $\mu \rightarrow e + \gamma$ for different VEV alignments of the triplet scalar flavon field involved in the model. To find the neutrino oscillation parameters we compare the light neutrino mass matrix obtained in our theory with the usual U_{PMNS} matrix (phenomenological) and obtain a set of six real equations. This set of equations are then solved to find the free parameters of neutrino mixing that are the lightest neutrino mass, Dirac CP phase δ and two Majorana masses α and β . This is done for different VEV alignments of triplet scalar flavon for both NH (Normal Hierarchy, when mass of third family is largest) and IH (Inverted Hierarchy, when mass of third family is smallest), out of which we select only the those vacuum alignments, that give values of light neutrino oscillation

parameters within their latest 3σ global best fit values. This data is next used as input to find the branching ratio of the $\mu \rightarrow e + \gamma$ decay for different allowed VEV alignments and analyze if the decay process can be predicted in the limit given by ongoing MEG collaboration [3] (bounds) or in the expected limit of upgraded MEG II experiment [4] (sensitivity).

2. Linear Seesaw under with $A_4 \times Z_5 \times Z_5'$ symmetry

There are several variants of seesaw mechanism which can be used to find the light neutrino masses and study their various implications such as cLFV decay, lepton number violating processes and leptogenesis. However, the low-scale seesaw mechanisms like the linear seesaw mechanism are more desirable as it is testable in the ongoing and future neutrino experiments. The neutrino mass matrix for linear seesaw mechanism [1] is given by :

$$M_\nu = \begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix} \text{-----(1)}$$

The light neutrino mass matrix after block diagonalisation as formulated by Deppischet. al. [2] is given by :

$$m_\nu = M_D^T M^{-1} M_L + \text{transpose} \text{-----(2)}$$

The heavy right handed neutrinos have masses $M_R = M$. The light neutrino masses can be realised in the presence of low-scale heavy Majorana neutrino as follows:

$$\left(\frac{m_\nu}{0.1eV}\right) = \left(\frac{M_D^T}{10GeV}\right) \left(\frac{M}{1TeV}\right)^{-1} \left(\frac{M_L}{10eV}\right) \text{-----(3)}$$

In the limit $M_L \rightarrow 0$, massless neutrino masses can be obtained. Hence this parameter is the monopole for the explicit lepton number violation. For our linear seesaw model we have incorporated A_4 flavor symmetry, and extend the SM with two singlet neutrinos - the right-

handed neutrino N_R and the sterile neutrino S_R . Apart A_4 flavour symmetry, we also use two additional Z_5 symmetries to construct our model. The particle content of our model is given by

Table 1 : Transformation of the fields under $A_4 \times Z_5 \times Z_5'$ symmetry for neutrino mass model realizing linear seesaw mechanism

	L	H	$e_{R\mu R\tau R}$			N	S	ϵ	Φ_T	Φ_s	η	ξ	τ	ρ
A 4	3	1	1	1''	1'	3	3	1	3	3	1	1'	1''	1
Z 5	ω	1	ω	$\omega\omega$		ω^2	ω^3	ω	1	ω^2	ω^2	ω^2	ω^2	1
Z 5'	ω	1	ω	$\omega\omega$		ω	ω^2		1	1	ω	$\omega\omega\omega\omega^2$		

The relevant Lagrangian of the neutrino sector is given by

$$L_\nu \supset Y_D \frac{\tilde{L}H N \epsilon^\dagger}{\Lambda} + Y_M N S \rho + Y_L \frac{\tilde{L}H S}{\Lambda} (\Phi_S^\dagger + \eta^\dagger + \xi^\dagger + \tau^\dagger) \text{-----(4)}$$

The flavour symmetry is broken spontaneously after the various scalars acquire non-zero VEV alignments as $\langle H \rangle = v_h, \langle \epsilon \rangle = v_\epsilon, \langle \rho \rangle = v_\rho, \langle \eta \rangle = v_\eta, \langle \xi \rangle = v_\xi, \langle \tau \rangle = v_\tau, \langle \Phi_S \rangle = v_s (\Phi_a,$

$\Phi_b, \Phi_c)$, without the loss of generality. All the couplings should be less than one (for the theory to be perturbative). From the Lagrangian it can be shown that

$$M_D = \frac{Y_D v_h v_\epsilon^\dagger}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{-----(5)}$$

$$M_L = \frac{Y_L v_h}{\Lambda} \begin{pmatrix} 2v_s^\dagger \Phi_a + v_\eta^\dagger & -v_s^\dagger \Phi_c + v_\tau^\dagger & -v_s^\dagger \Phi_b + v_\xi^\dagger \\ -v_s^\dagger \Phi_c + v_\tau^\dagger & 2v_s^\dagger \Phi_b + v_\xi^\dagger & -v_s^\dagger \Phi_a + v_\eta^\dagger \\ -v_s^\dagger \Phi_b + v_\xi^\dagger & -v_s^\dagger \Phi_a + v_\eta^\dagger & 2v_s^\dagger \Phi_c + v_\tau^\dagger \end{pmatrix} \text{-----(6)}$$

$$\text{and } M = Y_M v_\rho \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{-----(7)}$$

$$\Rightarrow m_\nu = F \begin{pmatrix} 2v_s^\dagger \Phi_a + v_\eta^\dagger & -v_s^\dagger \Phi_c + v_\tau^\dagger & -v_s^\dagger \Phi_b + v_\xi^\dagger \\ -v_s^\dagger \Phi_c + v_\tau^\dagger & 2v_s^\dagger \Phi_b + v_\xi^\dagger & -v_s^\dagger \Phi_a + v_\eta^\dagger \\ -v_s^\dagger \Phi_b + v_\xi^\dagger & -v_s^\dagger \Phi_a + v_\eta^\dagger & 2v_s^\dagger \Phi_c + v_\tau^\dagger \end{pmatrix} \text{-----(8)}$$

where, $F = \frac{2Y_L Y_D v_R^2 v_\epsilon^\dagger}{A^2 Y_M v_\rho}$ is a dimensionless constant. The (9x9) neutrino mass matrix can be block diagonalized by matrix K [5] which is expressed as

$$K^T M_\nu K = (M_\nu)_{diag} \text{-----(9)}$$

$$\Rightarrow K = W \cdot U' = \begin{pmatrix} \left(1 - \frac{1}{2} B^* B^T\right) & B^* \\ -B^T & \left(1 - \frac{1}{2} B^T B^*\right) \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} \text{-----(10)}$$

where U and V are unitary matrices that diagonalize the light and heavy neutrino masses. The matrix B* is a small perturbation which can be derived as

$$B^* = (M_D m^{T-1} \mu M^{-1}, M_D M^{T-1}) \approx O(10^{-11}, 10^{-2}) \text{-----(11)}$$

The matrix U is the usual PMNS matrix and matrix V can be numerically computed as [6]

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} I + \frac{\mu M^{-1}}{4} & I - \frac{\mu M^{-1}}{4} \\ -I + \frac{\mu M^{-1}}{4} & I + \frac{\mu M^{-1}}{4} \end{pmatrix} \text{-----(12)}$$

To compute the matrix M_D, we take the help of Casas-Ibarra parametrisation which for linear seesaw [6] is given as:

$$M_D = U m_n^{1/2} R m_n^{1/2} U^T M_L^{T-1} M^T \text{-----(13)}$$

where $m_n = \text{diag}(m_1, m_2, m_3)$, $m_{1,2,3}$ being the light neutrino masses. The matrix R satisfies the condition $R + R^T = 1$ [7] which can be written in the general form

$$R = \begin{pmatrix} 1/2 & a & b \\ -a & 1/2 & c \\ -b & -c & 1/2 \end{pmatrix} \text{-----(14)}$$

The parameter a,b,c are of the order such that M_D is the order of 0.1 eV.

3. cLFV (charged lepton flavour violation) decay of $\mu \rightarrow e + \gamma$

The existence of cLFV decay is one of the most popular predictions of neutrino mass models. The most commonly studied LFV process is $l_\alpha \rightarrow l_\beta + \gamma$. The study of this process over the

years have contributed to experimental developments with increased sensitivities and bounds as well, as these processes are expected to possess the highest rates. The $\mu \rightarrow e + \gamma$ decay observables have the best experimental limit of $BR(\mu \rightarrow e + \gamma) < 4.2 \times 10^{-13}$ [3] given by

MEG(Mu to E Gamma) collaboration. The upgrade to MEG II is expected to improve the bound upto $< 5 \times 10^{-14}$ [4]. In this section, we compute the bounds of the cLFV contributed by

$$BR(l_\alpha \rightarrow l_\beta \gamma) \approx \frac{\alpha_W^3 \sin^2 \theta_W m_{l\alpha}^5}{256 M_W^4 \Gamma_{l\alpha}} \left| \sum_{i=1}^9 K_{\alpha i}^* K_{\beta i} G\left(\frac{m_i^2}{M_W^2}\right) \right|^2, \tag{15}$$

$$\text{where, } G(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^2}{2(1-x)^4} \ln x \tag{16}$$

Here $x = (m_i^2 / M_W^2)$, $\alpha_W = g^2 / 4\pi$, where g is the weak coupling, θ_W being the electroweak mixing angle, M_W is the W^\pm boson mass, m_i as the mass of both active and sterile neutrinos ,

RESULT AND CONCLUSION

We have computed the BR of $(\mu \rightarrow e + \gamma)$ process for six allowed case of vacuum

linear seesaw model using the parameter values obtained in the previous section. The most general form of the branching ratio of $l_\alpha \rightarrow l_\beta + \gamma$ is given by [6], [8],[9]

$m_{l\alpha}$ as the mass of the decaying charged lepton l_α and $\Gamma_{l\alpha}$ as the total decay width of the decaying charged lepton l_α .

alignment applied on triplet scalar flavon field whose values are shown in the Table below:

Table 2: The upper and lower limits of branching ratio BR($\mu \rightarrow e + \gamma$) as computed in our analysis for different allowed VEVs of triplet flavon, in linear seesaw model

Sl. No.	VEV	Hierarchy	Range of BR ($\mu \rightarrow e + \gamma$)
1.	(0,1,1) / (0,-1,-1)	NH	$(7.28147 \times 10^{-16}, 2.99351 \times 10^{-15})$
2.	(-1,1,1) / (1,-1,-1)	NH	$(2.69891 \times 10^{-16}, 8.5382 \times 10^{-16})$
3.	(0,1,-1) / (0,-1,1)	IH	$(1.16469 \times 10^{-14}, 1.67044 \times 10^{-14})$

It is seen that the BR($\mu \rightarrow e + \gamma$) for the allowed case in IH satisfies the current experimental bound of MEG i.e., $< 4.2 \times 10^{-13}$ whereas the values of the BR($\mu \rightarrow e + \gamma$) for the two NH cases satisfies the future expected sensitivity of MEG II experiment, i.e., $< 5 \times 10^{-14}$. Also, it is interesting to observe from our results above that we can pinpoint the mass hierarchy for

5. Appendix : A4 product rule

The quark and lepton masses, mixing and CP violation can be described using a discrete

different allowed VEV alignments. We have done this computation for a tolerance of $\leq 10^{-5}$ which gives us a very precise result. This model would be testable in the next-generation experiments giving important implications on cLFV processes, mass hierarchy, Octant degeneracy and baryon asymmetry of the universe (BAU, through Leptogenesis).

non-abelian group of even permutations of four objects called A_4 flavour symmetry, the same symmetry group can also be represented

by a tetrahedron. It consist of four irreducible representations - three one dimensional (singlets, **1**) and one three dimensional

$$\begin{aligned} 1 \times 1 &= 1, & 1'' \times 1'' &= 1', \\ 1' \times 1' &= 1'', & 3 \times 3 &= 1 + 1' + 1'' + 3_s + 3_a. \end{aligned}$$

(triplet, **3**) respectively [10]. The product rule of A_4 symmetry can be represented as :

Acknowledgments - We thank RUSA grant of Govt. of India, for funds to set up a sophisticated computing cluster facility in the Computer laboratory of our Department, where the computational results presented in this work were done.

REFERENCE

- [1] M. Malinsky, J. C. Romao and J. W. F. Valle, Phys. Rev. Lett. **95** (2005), 161801
doi:10.1103/PhysRevLett.95.161801
[arXiv:hep-ph/0506296 [hep-ph]].
- [2] F. F. Deppisch, L. Graf, S. Kulkarni, S. Patra, W. Rodejohann, N. Sahu and U. Sarkar, Phys. Rev. D **93** (2016) no.1, 013011
doi:10.1103/PhysRevD.93.013011
[arXiv:1508.05940 [hep-ph]].
- [3] A. M. Baldini *et al.* [MEG], Eur. Phys. J. C **76** (2016) no.8, 434
doi:10.1140/epjc/s10052-016-4271-x
[arXiv:1605.05081 [hep-ex]].
- [4] P. W. Cattaneo [MEG II], JINST **12** (2017) no.06, C06022
doi:10.1088/1748-0221/12/06/C06022
[arXiv:1705.10224 [physics.ins-det]].
- [5] S. Blanchet, P. S. B. Dev and R. N. Mohapatra, Phys. Rev. D **82** (2010), 115025
doi:10.1103/PhysRevD.82.115025
[arXiv:1010.1471 [hep-ph]].
- [6] B. Karmakar and A. Sil, Phys. Rev. D **96** (2017) no.1, 015007
doi:10.1103/PhysRevD.96.015007 [arXiv:1610.01909 [hep-ph]].
- [7] M. J. Dolan, T. P. Dutka and R. R. Volkas, JCAP **06** (2018), 012
doi:10.1088/1475-7516/2018/06/012
[arXiv:1802.08373 [hep-ph]].
- [8] D. V. Forero, S. Morisi, M. Tortola and J. W. F. Valle, JHEP **09** (2011), 142
doi:10.1007/JHEP09(2011)142
[arXiv:1107.6009 [hep-ph]].
- [9] A. Ilakovac and A. Pilaftsis, Nucl. Phys. B **437** (1995), 491
doi:10.1016/0550-3213(94)00567-X
[arXiv:hep-ph/9403398 [hep-ph]].
- [10] A. Abada and M. Lucente, Nucl. Phys. B **885** (2014), 651-678
doi:10.1016/j.nuclphysb.2014.06.003
[arXiv:1401.1507 [hep-ph]].
- [11] H. Ishimori, T. Kobayashi, H. Ohki,

- H. Okada, Y. Shimizu and M. Tanimoto, Lect. Notes Phys. **858** (2012), 1-227 doi:10.1007/978-3-642-30805-5
- [12] C.M. Thakar, S.S. Parkhe, A. Jain *et al.*, 3d Printing: Basic principles and applications, Materials Today: Proceedings, <https://doi.org/10.1016/j.matpr.2021.06.272>
- [13] Khan, R. M. I., Kumar, T., Supriyatno, T., & Nukapangu, V. (2021). The Phenomenon of Arabic-English Translation of Foreign Language Classes During The Pandemic. *IjazArabi Journal of Arabic Learning*, 4(3). <https://doi.org/10.18860/ijazarabi.v4i3.13597>
- [14] Sajja, G., Mustafa, M., Phasinam, K., Kaliyaperumal, K., Ventayen, R., & Kassanuk, T. (2021). Towards Application of Machine Learning in Classification and Prediction of Heart Disease. 2021 Second International Conference On Electronics And Sustainable Communication Systems (ICESC). <https://doi.org/10.1109/icesc51422.2021.9532940>
- [15] Veluri, R., Patra, I., Naved, M., Prasad, V., Arcinas, M., Beram, S., & Raghuvanshi, A. (2021). Learning analytics using deep learning techniques for efficiently managing educational institutes. *Materials Today: Proceedings*. <https://doi.org/10.1016/j.matpr.2021.11.416>