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# THE (4, ($C_1$, $C_2$)) - REGULAR INTUITIONISTIC FUZZY GRAPH ON SOME SPECIAL GRAPHS

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**ABSTRACT**

This paper is to defined (4, ($c_1$, $c_2$)) - regular intuitionistic fuzzy Graphs on a cycle of length ≥ 5($c_n$ for n ≥ 5), Wagner graph and Barbell graph ($B_{2,2}$). In addition, some more properties and examples of these graphs are studied and viewed for total (4, ($c_1$, $c_2$)) - regular intuitionistic fuzzy graphs.

**Key words: (4,($c_1, c_2$)) - regular and totally (4,($c_1, c_2$)) - regular intuitionistic fuzzy graph,cycle of length ≥5, Wagner graph and Barbell graph**

## 1. INTRODUCTION

Some properties of fuzzy graphs [FGs] was introduced by Azriel Rosenfeld **[3]**. Shanmuga Raj , Chandrasekar and Meena Devi **[4]** developed and discussed some various properties of (4, k) - regular and totally (4, k) – regular fuzzy graphs. Already, we have introduced (3, ($c_1, c_2$)) – regular intuitionistic fuzzy graphs **[5]** and (4,($c_1, c_2$)) – regular intuitionistic fuzzy graphs **[6]**. The concept of (3,($c_1, c_2$)) – regular intuitionistic fuzzy graphs and (4,($c_1, c_2$)) – regular intuitionistic fuzzy graphs introduced by Suja kumari and Chithamparathanu pillai. In this paper, we defines (4,($c_1, c_2$)) - regular and totally (4,($c_1, c_2$)) - regular intuitionistic fuzzy graphs on some special graphs such as cycle of length ≥

5,Wagner graph and Barbell graph.The condition that provide equivalent connection between these two graphs and a characterization of the (4,($c_1$,$c_2$)) - regular intuitionistic fuzzy graphs on a cycle of length ≥ 5,Wagner graph and Barbell graph are exhibited.

- The following terminologies fuzzy graph, intuitionistic fuzzy graph and regular intuitionistic fuzzy graph are respectively abbreviated as FG, IFG and RIFG.

## 2. Preliminaries:

Let G = (A, B) be an IFG. The $d_4$ – degree of a vertex v in V [6] is defined as $d_4(v) = (d_{4\mu}(v), d_{4\gamma}(v))$……(1) where $d_{4\mu}(v) = \{\sum\mu_2^4(v\ w) : \mu_2^4(v\ w) = \sup\{\mu_2(v\ v_1) \Lambda \mu_2(v_1\ v_2) \Lambda \mu_2(v_2, v_3) \Lambda \mu_2(v_3, w)\ \forall\ v, v_1, v_2, v_3, w \in V\}$ and $d_{4\gamma}(v) = \{\sum\gamma_2^4(v\ w) : \gamma_2^4(v\ w) = \inf[\gamma_2(v\ v_1)\ V\ \gamma_2(v_1\ v_2)\ V\ \gamma_2(v_2\ v_3)\ V\ \gamma_2(v_3, w)]\ \forall\ v, v_1, v_2, v_3, w \in V\}$ and the total $d_4$ – degree of a vertex v in V [6] is $td_4(v) = (td_{4\mu}(v), td_{4\gamma}(v))$……(2) where $td_{4\mu}(v) = d_{4\mu}(v) + \mu(v)$ and $td_{4\gamma}(v) = d_{4\gamma}(v) + \gamma(v)$.

## 3. The (4,($c_1, c_2$)) - RIFG and totally (4,($c_1, c_2$)) - RIFG:

**Definition 3.1** If $d_4(u) = (c_1, c_2)$ ∀u ∈ V then the graph G [6] is called (4, ($c_1$, $c_2$)) –RIFG and if each vertex of G has same total $d_4$ – degree [6] , then G is called totally (4, ($c_1$, $c_2$)) - RIFG.

We are giving suitable examples for above definitions

**Example 3.2** Let us consider an IFG,G on $G^*$ in **Figure 1**

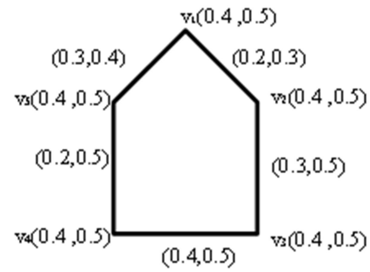


**Figure: 1**

Here, $d_4(v) = (0.4, 1.0)$ for all v = $v_1$ ,$v_2$ ,$v_3$ ,$v_4$, $v_5$∈V and $td_4(v) = (0.8, 1.5)$ for all v ∈ V.

## 4. The (4,($c_1, c_2$))- RIFGs on a cycle of length≥5

**Theorem 4.1**

If an IFG, G: (A,B) on $G^*: (V, E)$is a cycle of length ≥5 and B is a constant function , then G is a(4,2($c_1, c_2$)) - RIFGs.

**Proof:** Assume, B is a constant function. Take B(uv) = ($c_1, c_2$) say , for all uv∈ $E$.

Now, the $d_4$- degree of v ∈V in G = ($2c_1, 2c_2$).

∴ $d_4(v)$ = 2($c_1, c_2$) for all v∈.

Thus the graph G is a (4,2($c_1, c_2$)) - RIFGs

**Remark:4.2:** If an IFG, G on $G^*$is a cycle of length ≥5 and G is a (4,2($c_1, c_2$) - RIFGs, then B is not a constant function.

**Example: 4.3** Consider an IFG,G on $G^*$ is a cycle of length 7 in **Figure 2**

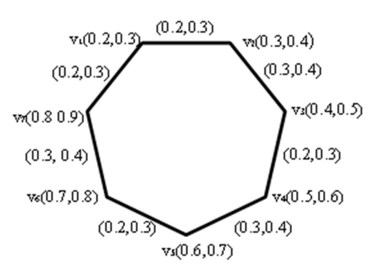


**Figure: 2**

Here $d_4(v_1)$ = (0.4 ,0.8) , $d_4$ ($v_2$) = (0.4 ,0.8) , $d_4(v_3)$ = … $d_4(v_7)$ = (0.4 ,0.8).This graph G is a (4, ($c_1$,$c_2$ ))- RIFG. But B is not a constant function .

**Remark : 4.4** If G is an IFG on$G^*$ is a cycle of length ≥5 and B is a constant function, then G is not a totally (4,($c_1$,$c_2$)) - RIFGs.

**Theorem: 4.5**

If G is a FG on $G^*$ is an even cycle of length 6 and the alternate edges have same positive and negative membership values, then G is a (4,($c_1, c_2$)) - RIFGs**.**

**Proof:**Assume that alternate edges have same membership values then

$$B(e_i) = \begin{cases} (c_1, c_2) & \text{if i is odd} \\ (c_3, c_4) & \text{if i is even} \end{cases}$$

Now, to find the value of $d_4(v_i)$ for the following four possible cases:

1. If $c_1 > c_3$ and$c_2 > c_4$, 2.If $c_1 > c_3$ *and* $c_2 < c_4$

3. If $c_1$<$c_3$ and $c_2 < c_4$, 4.If $c_1 < c_3$ and $c_2 > c_4$

In all the cases , $d_4(v_i)$ = constant (say ) ( $c_1$ , $c_2$) , for all $v_i \in V$.

Thus the graph G is a (4 ,( $c_1$,$c_2$)) -RIFG.

**Remark : 4.6** The above theorem 4.5 is not true for totally (4,($c_1, c_2$))– RIFGs**.**

**Example: 4.7** Let us consider an IFG, G on $G^*$ is a cycle of length 6 in **Figure 3**

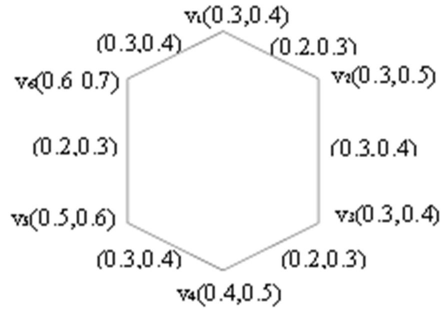


**Figure:3**

$td_4(v_1)$ = (0.7, 1.2), $td_4(v_2)$ = (0.7, 1.3), $td_4(v_3)$ = (0.7, 1.2), $td_4(v_4)$ = (0.8, 1.3), $td_4(v_5)$ = (0.9, 1.4), $td_4(v_6)$ = (1.0 ,1.5). This graph G is not a totally (4, ($c_1$ , $c_2$)) – RIFGs**.**

**Theorem : 4.8**

If G is an IFG on $G^*$ is an even cycle of length ≥ 6 and all the vertices are same positive and same negative membership values , then theorem 4.5 is hold for totally (4 ,( $c_1, c_2$))- RIFGs**.**

**Theorem 4.9**

Let G : (A,B ) be an IFG on $G^*$: (V,E) is an odd cycle $c_n$, $n \geq 5$ and the alternate edges have same membership values , then G is a (4, ($c_1, c_2$)) – RIFG.

**Example:4.10** Let us consider an IFG, G on $G^*$ is a cycle of length 5 in **Figure 4**

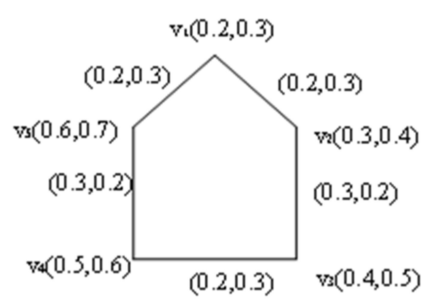


**Figure 4**

Here, $d_4(v_1)$ = (0.4, 0.6), $d_4(v_2)$ = (0.4, 0.6), $d_4(v_3)$ = (0.4, 0.6), $d_4(v_4)$ = (0.4, 0.6), $d_4(v_5)$ = (0.4 ,0.6). So G is (4,(0.4 ,0.6) - RIFG.

**Theorem :4.11**

If an IFG, G on $G^*$ is an odd cycle $c_n$, $n \geq 5$ and alternate edges have same positive and negative membership values . Even if A is not a constant function , then G is not a totally (4,($c_1$,$c_2$))- RIFG.

For example, From the above figure : 4, Here, $td_4(v_1)$ = (0.6 ,0.9) , $td_4(v_2)$ = (0.7 ,1.0) , $td_4(v_3)$ = (0.8 ,1.1) , $td_4(v_4)$ = (0.9 ,1.2) , $td_4(v_5)$ = (1.0,1.3). This graph G is not a totally (4,($c_1$, $c_2$)) - RIFG. But A is not a constant function.

**Theorem 4.12**

Let G : (A,B ) be an IFG on $G^*$: (V,E) is any cycle $c_n$, $for$ n > 4. Let, $B(e_i)$= $\begin{cases}(k_1, k_2) & \text{if i is odd}\\(k_3, k_4) & \text{if i is even}\end{cases}$ and $k_3 \geq k_1$ ,$k_4 \geq k_2$ then G is a (4, ($c_1$ , $c_2$)) − RIFGs.

**Proof :Case :1** If $G^*$ is an even cycle , then $e_1, e_2, \dots, e_{2n}$ be the edges of $G^*$

Now, $d_4(v_1) = (\mu_2^{(4)}(e_1) \Lambda \mu_2^{(4)}(e_2) \Lambda \mu_2^{(4)}(e_3) \Lambda \mu_2^{(4)}(e_4),\ \gamma_2^{(4)}(e_1) \vee \gamma_2^{(4)}(e_2) \vee \gamma_2^{(4)}(e_3) \vee \gamma_2^{(4)}(e_4)) + (\mu_2^{(4)}(e_{2n}) \Lambda \mu^{(4)}(e_{2n-1}) \Lambda \mu_2^{(4)}(e_{2n-2})) \Lambda \mu_2^{(4)}(e_{2n-3}),\ \gamma_2^{(4)}(e_{2n}) \vee \gamma_2^{(4)}(e_{2n-1}) \vee \gamma_2^{(4)}(e_{2n-2}) \vee \gamma_2^{(4)}(e_{2n-3}))$

$= (k_1 \Lambda k_3 \Lambda k_1 \Lambda k_3, k_2 \vee k_4 \vee k_2 \vee k_4) + (k_3 \Lambda k_1 \Lambda k_3 \Lambda k_1, k_4 \vee k_2 \vee k_4 \vee k_2)$

$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4) = (c_1, c_2)$ where $(c_1, c_2) = (2k_1, 2k_4)$

Similarly, $d_4(v_2) = (c_1, c_2)$ where $(c_1, c_2) = (2k_1, 2k_4)$ …$d_4(v_i) = (c_1, c_2)$ where $(c_1, c_2) = (2k_1, 2k_4)$ for i = 1, 2, 3,…..2n

**Case: 2:** If $G^*$ is an odd cycle, then $e_1, e_2, \dots, e_{2n+1}$ be the edges of $G^*$

Now, $d_4(v_1) = (\mu_2^{(4)}(e_1) \Lambda \mu_2^{(4)}(e_2) \Lambda \mu_2^{(4)}(e_3) \Lambda \mu_2^{(4)}(e_4),\ \gamma_2^{(4)}(e_1) \vee \gamma_2^{(4)}(e_2) \vee \gamma_2^{(4)}(e_3) \vee \gamma_2^{(4)}(e_4)) + (\mu_2^{(4)}(e_{2n-2}) \Lambda \mu_2^{(4)}(e_{2n-1}) \Lambda \mu_2^{(4)}(e_{2n}) \Lambda \mu_2^{(4)}(e_{2n+1})), (\gamma_2^{(4)}(e_{2n-2}) \vee \gamma_2^{(4)}(e_{2n-1}) \vee \gamma_2^{(4)}(e_{2n}) \vee \gamma_2^{(4)}(e_{2n+1})))$

$= (k_1 \Lambda k_3 \Lambda k_1 \Lambda k_3, k_2 \vee k_4 \vee k_2 \vee k_4) + (k_1 \Lambda k_3 \Lambda k_1 \Lambda k_3, k_2 \vee k_4 \vee k_2 \vee k_4)$

$= (k_1, k_4) + (k_1, k_4) = (2k_1, 2k_4) = (c_1, c_2)$, where $(c_1, c_2) = (2k_1, 2k_4)$

$d_4(v_2) = (c_1, c_2)$, where $(c_1, c_2) = (2k_1, 2k_4)$ …$d_4(v_i) = (c_1, c_2)$, where $(c_1, c_2) = (2k_1, 2k_4)$ for i = 1, 2, 3,….2n + 1

Hence, G is (4, ($c_1$, $c_2$)) – RIFG,where $(c_1, c_2) = (2k_1, 2k_4)$.

**Remark 4.13** Let G = (A, B) be an IFG on $G^*$(V, E) is any cycle $c_n$ for n > 4 and all the vertices are same positive and negative membership values. Let, $B(e_i)$ = $\begin{cases}(k_1, k_2) & \text{if i is odd}\\(k_3, k_4) & \text{if i is even}\end{cases}$ and $k_3 \geq k_1$, $k_4 \geq k_2$ then G is a totally (4, ($c_1$, $c_2$)) - RIFG.

**Theorem 4.14**

Let G :(A,B ) be an IFG on $G^*$: (V,E) is any cycle $c_n$, $for$ n > 4. Let, $B(e_i)$ = $\begin{cases}(k_1, k_2) & \text{if i is odd}\\(k_3, k_4) & \text{if i is even}\end{cases}$ and $k_3 \leq k_1$ ,$k_4 \leq k_2$ then G is a (4, ($c_1$ , $c_2$)) − RIFG.

**Proof :Case :1** If $G^*$ is an even cycle, then $e_1, e_2, \dots, e_{2n}$ be the edges of $G^*$

Now, $d_4$ $(v_1)$ = $(\mu_2^{(4)}(e_1)$ $\Lambda\mu_2^{(4)}(e_2)$ $\Lambda\mu_2^{(4)}(e_3)\Lambda\mu_2^{(4)}(e_4)$, $\gamma_2^{(4)}(e_1)$ V $\gamma_2^{(4)}(e_2)$ V $\gamma_2^{(4)}(e_3)$ V $\gamma_2^{(4)}(e_4))$ + $(\mu_2^{(4)}(e_{2n})\Lambda\mu_2^{(4)}(e_{2n-1})\Lambda\mu_2^{(4)}(e_{2n-2})\Lambda\mu_2^{(4)}(e_{2n-3})$ , $\gamma_2^{(4)}(e_{2n})$ V $\gamma_2^{(4)}(e_{2n-1})$ V $\gamma_2^{(4)}(e_{2n-2})$ V $\gamma_2^{(4)}(e_{2n-3}))$

= $(k_1\Lambda\ k_3\Lambda\ k_1\Lambda\ k_3,\ k_2$ V $k_4$ V $k_2$ V $k_4)$ + $(k_3\Lambda\ k_1\Lambda\ k_3\Lambda\ k_1,\ k_4$ V $k_2$ V $k_4$ V $k_2)$

= $(k_3,\ k_2)$ + $(k_3,\ k_2)$ = $(2k_3,\ 2k_2)$ = $(c_1,\ c_2)$ where $(c_1, c_2) = (2k_3, 2k_2)$

Similarly, $d_4$ $(v_2)$= $(c_1,\ c_2)$ where $(c_1,\ c_2)$ = $(2k_3,\ 2k_2)$ … $d_4$ $(v_i)$ = $(c_1,\ c_2)$ where $(c_1,\ c_2)$ = $(2k_3,\ 2k_2)$, for i = 1, 2, 3, …2n

**Case: 2:** If $G^*$ is an odd cycle, then $e_1$, $e_2$,……. $e_{2n+1}$ be the edges of $G^*$

Now, $d_4$ $(v_1)$ = $(\mu_2^{(4)}(e_1)$ $\Lambda\mu_2^{(4)}(e_2)$ $\Lambda\mu_2^{(4)}(e_3)$ $)\Lambda\mu_2^{(4)}(e_4)$, $\gamma_2^{(4)}(e_1)$ V $\gamma_2^{(4)}(e_2)$ V $\gamma_2^{(4)}(e_3)$ V $\gamma_2^{(4)}(e_4))$ + $(\mu_2^{(4)}(e_{2n-2})$ $\Lambda\mu_2^{(4)}(e_{2n-1})\Lambda\mu_2^{(4)}(e_{2n}$ $)\Lambda\mu_2^{(4)}(e_{2n+1})$, $\gamma_2^{(4)}(e_{2n-2}$ $)\gamma_2^{(4)}(e_{2n-1})$V $\gamma_2^{(4)}(e_{2n}$ $)$ V $\gamma_2^{(4)}(e_{2n+1})$

= $(k_1\Lambda\ k_3\Lambda\ k_1\Lambda\ k_3,\ k_2$ V $k_4$ V $k_2$ V $k_4)$ + $(k_1\Lambda\ k_3\Lambda\ k_1\Lambda\ k_3,\ k_2$ V $k_4$ V $k_2$ V $k_4)$

= $(k_3,\ k_2)$ + $(k_3,\ k_2)$ = $(2k_3,\ 2k_2)$ =$(c_1,\ c_2)$, where $(c_1, c_2) = (2k_3, 2k_2)$

Similarly, $d_4(v_2)$ = $(c_1,\ c_2)$, where $(c_1,\ c_2)$ = $(2k_3,\ 2k_2)$ … $d_4$ $(v_i)$ = $(c_1,\ c_2)$, where $(c_1,\ c_2)$ = $(2k_3,\ 2k_2)$, for i = 1, 2, 3,…., 2n + 1

Hence, G is (4, $(c_1,\ c_2)$) – RIFGs, where $(c_1,\ c_2) = (2k_3,\ 2k_2)$.

**Remark 4.15**Let G = (A, B) be an IFG on $G^*$(V, E) is any cycle $c_n$ for n > 4 and all the vertices are same positive and negative membership values. Let, $B(e_i)$ = $\begin{cases} (k_1, k_2) & \text{if i is odd} \\ (k_3, k_4) & \text{if i is even} \end{cases}$ and $k_3 \leq k_1$, $k_4 \leq k_2$ then G is a totally (4, $(c_1,\ c_2)$) – RIFG.

## 5. The (4,$(c_1, c_2)$) – RIFGs on Wagner graph with specific membership functions:

**Theorem :5.1**

Let G: (A,B) be an IFG on $G^*: (V, E)$ is a Wagner graph and B is a constant function, then G is a (4, 5$(c_1, c_2)$) - RIFG.

**Proof:**Assume that B is a constant function. Take B(uv) = $(c_1, c_2)$ say, for all uv ∈ E

Then $d_4$- degree of v ∈ V in G =5$(c_1, c_2)$. (ie) $d_4(v)$ = 5 $(c_1, c_2)$) , for all v ∈ V. Hence the graph is a (4,5$(c_1, c_2)$)- RIFG.

**Theorem 5.2**

If G is an IFG on $G^*$ is a Wagner graph and G is a (4,( $c_1, c_2$))- RIFG, then B is not a constant function

**Example: 5.3**Consider G = (A, B)be an IFG such that G*: (V ,E) is Wagner graph in **Figure 5**

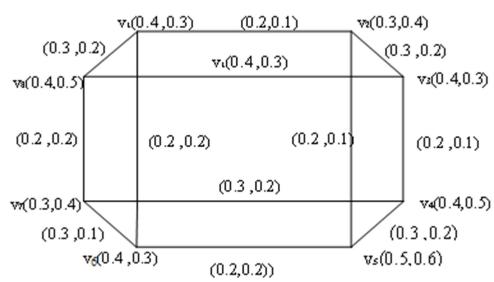


**Figure: 5**

Here $d_4(v_1)$ = (1.0 ,1.0) , $d_4(v_2)$ = (1.0 ,1.0), $d_4(v_3)$ = (1.0 ,1.0) , $d_4(v_4)$ = (1.0 ,1.0), $d_4(v_5)$ = (1.0 ,1.0), $d_4(v_6)$ = (1.0 ,1.0), $d_4(v_7)$ = (1.0 ,1.0) , $d_4(v_8)$ =(1.0 ,1.0). This graph G is a (4,

$(c_1, c_2)$) - RIFG. But B is not a constant function.

**Remark 5.4:** Above theorem 5.1 is not hold for totally (4,$(c_1, c_2)$) – RIFGs.

## 6. The (4,$(c_1, c_2)$) – RIFG on Barbell graph,$B_{n,n}$(n>1) with specific membership function:

**Theorem 6.1**

If G : (A,B) be an IFG on G*: (V,E) is a Barbell graph $B_{n,n}$of order 2n and B is a constant function , then G is (4,( $c_1 c_2$))- RIFG, where $(c_1,c_2)$ = n $(\mu_2^{(4)}, \gamma_2^{(4)})$(uv) , uv∈E.

**Theorem 6.2**

If G : (A,B) be an IFG on $G^*: (V, E)$ is a Barbell graph and G is (4,$(c_1, c_2)$), where $(c_1, c_2) =$ n $(\mu_2^{(4)}, \gamma_2^{(4)})(uv), uv \in E$ ,RIFG, then B is not a constant function.

**Example:6.3** Let us consider an IFG, G on $G^*$in **Figure 6**

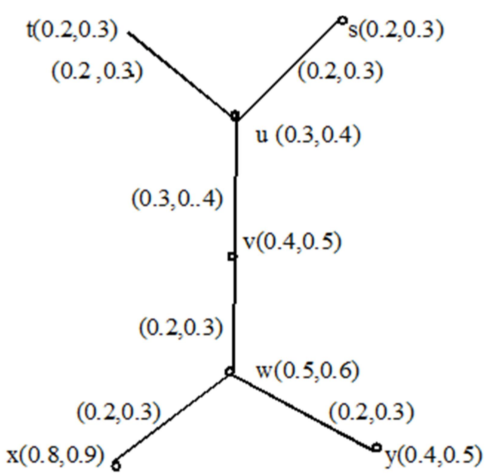


**Figure 6**

Here $d_4(u)$ =(.4,.8) for all u = t ,s, u, v, w, x, y ∈ V. Hence this graph G is a (4, ( .4,.8)) RIFG. But B is not a constant function.

**Theorem :6.4**

If G is a FG on $G^*$ is a Barbell graph ( $B_{n,n}$, n >1 ) and the pendent edge has positive membership value less than the membership value of middle edge and negative membership value greater than the negative membership value of middle edge then G is (4, n $(c_1,c_2)$) - RIFG. Where $(c_1, c_2)$ is the positive and negative membership value of pendent edges.

**Remark : 6.5:** Let G : (A,B) be an IFG on $G^*$: (V,E) is a Barbell graph ($B_{n,n}$ , n > 1) and the membership values of A is not a constant function , then the above theorem 6.1 and 6.2 are not totally (4, $(c_1, c_2)$) - RIFGS.

## CONCLUSION

In this paper, (4, $(c_1, c_2)$)–RIFGs and totally ( 4,( $c_1, c_2$)) - RIFGs are compared through some special graphs such as Cycle of length ≥ 5 ,Wagner graph and Barbell graph . Some various properties and examples of (4, $(c_1, c_2)$) - RIFGs are studied through these special graphs and the results are examined for totally (4,$(c_1, c_2)$) -RIFGs.

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