



**International Journal of Biology, Pharmacy  
and Allied Sciences (IJBPAS)**

*'A Bridge Between Laboratory and Reader'*

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## **THE ROLE OF FRACTAL GEOMETRY AND CHAOS PHENOMENON IN MODERN ARCHITECTURE**

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### **ABSTRACT**

This paper attempts to show studies on fractal geometry and chaos phenomenon in architecture and its relationship with the mathematics science which has generated modern window in contemporary architecture. After proposing butterfly effect theory by Edward Lorentz, the chaos math and chaos science gradually became proposed. Benoit Mandelbrot first proposed math chaos in 1975. Mandelbrot set is the most complex fractal which a function of one of the easiest mathematical rules. chaos rules are obtained in the infinite limit of a simple mathematical formula. According to his theory, simple rules create complex shapes. Fractals are shapes that are unlike the shape of Euclidean geometry and they are not regular at all. First, these figures are irregular; second, their disorder is identical in all scales. With respect to existing forms in the nature, it is obvious that Euclidean geometry cannot explain and describe complex shapes and seemingly, it is not naturally irregular. The amount of disorder is identical in all scales. Fractal object is seen identical from far and near, otherwise, it is self-similar. When we approach a fractal object, we see that small pieces of it, which seem amorphous grains in distance, will become a certain shape, which is less than its shape and more than same form that from a distance. Many examples of fractals can be seen in nature.

The aim of this paper is to implement a different perspective of the architecture with respect to the environmental assessments in fractal form in order to defend the sustainability of climate architecture and can offer a new style of architecture, and that with such an approach, what kind of information can be obtained.

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**Keywords:** Fractal geometry, chaos phenomenon, Euclidean geometry, traditional texture, fractal structures

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## 1. INTRODUCTION

Euclidean geometry only investigates available classic forms in the nature. In this geometry of the shapes and uneven functions, turbulent and non-classical had no place because of their uncontrollability. Finally, in 1994, the mascot of one of the mathematical theories proposed in 1897 was broken and *Mandel Brot*, Polish mathematician, became the founder of a new geometry which is called sizeless geometry or fractal geometry is one of new branches of mathematics that has shown unique flexibility against the interpretation and simulation of various forms. By using fractal geometry, a clear horizon has been placed in front of mathematicians and researchers in the field of retelling the behavior of the functions and seemingly uneven and chaotic sets.

Fractal geometry or the geometry of fractals is a phenomenon that had recently entered the world of mathematics. Before Mandelbrot coined the term, for such forms, the term "monster curve" was used. The term fractal is derived from the Latin word *Fractus* meaning that a stone broken and crushed irregularly, which was introduced into the world of mathematics by the Polish mathematician in 1976. During the 1990s, a

number of experts have developed this concept in relation to vernacular architecture. The principle of this theory in relation to vernacular architecture has been provided in two cases, one theoretical and the other form calculation. Theoretical discussion provided and developed this kind of architecture and the first set of local structures analysis was carried out using computer software (Salingaros), irregular textures got quantifiable and measurable with the help of fractal geometry. In the present article of Carl Bovill, we are going to conduct a study on the effects of the local architecture with the fractal nature.

## 2. The Phenomenon of Chaos

After two theories of relativity and quantum, the theory of final order is the third scientific revolution of our time. Theory of relativity disturbed Newtonian theory of time, and absolute space and quantum theory disturbed the dream of accurate, controllable and sound measurements and the final order theory questioned certain predictions that stemmed from the idea of Laplace.

Chaos by revealing the relationship between simplicity and complexity, and the relationship between regular and random

connects our every-day experience and the laws of nature as well.

Chaos presents a world that while it is deterministic and follows basic laws of physics and it may be chaotic, complex and unpredictable. Chaos shows that predictability is a rare phenomenon that acts only within the constraints of science which is extracted from the heart of rich diversity of our complex world. In addition, it provides the ability of simplifying complex phenomena and doubts in the traditional procedures of patterning.

For the first time, Edward Lorentz, Professor of Meteorology at the M.I.T University of the U.S. proposed it in the early 70's. The term chaos can be seen in many of the articles that have been published in recent years. Before Edward Lorentz proposes chaos theory, thinkers imagined that all the phenomena of the universe are random or deterministic, but Lorentz proposed a theory, which suggested that as a result, some of the phenomena of the world and perhaps many of them had a random face but they follow very complex rules.

### 3. Chaos Definition

Chaos has been defined in various ways. These are a few examples of these definitions:

- a kind of order without periodicity

- Seemingly random and repetitive behavior in a simplified and deterministic system (e.g., hours)
- A qualitative study of unstable non-cyclic behavior in non-linear dynamic deterministic systems

And now another definition of Yan Stewart, a mathematician who studies in this field: "Chaos is the ability of simple patterns with no internal random characters, in presenting a very irregular behavior".

Therefore, disorder and chaos are not regarded as the concept of non-organization, inefficiency, and and chaos but disorder is the existence of unpredictable and random aspects in dynamic phenomena which has its own properties. Disorder and chaos are a kind of final order in chaos. anarchy derby. Disorder and chaos are a kind of orderly disorder or order in chaos. It is chaotic, since the results are unpredictable and it is regular since it enjoys a kind of certainty.

Many scientists believe that "chaos" is a poor name for this new science, because it brings to mind the randomness. In their view, the most important message of this theory is that, in nature, simple processes can make great structures of this complexity without really being random.

### 4. Chaos Properties

#### 4.1. Feedback

Feedback, is the property of the systems its result or output affects the system input, and thus alters its function.

Oscillatory systems are chaotic because there is a feedback element. Chaotic behavior is the result of nonlinear forces that have returned on their own called nonlinear feedback, which is a basic prerequisite chaotic.

#### 4.2. Dependency on initial conditions (*Butterfly Effect*)

Another unique characteristic of chaotic systems is their sensitive dependency on initial conditions. extremely initial small changes lead to greater changes. This behavior is called the signature of chaos.

Lorenz theory's butterfly effect is in such a way that a slight change at the beginning leads to a large change at the end of the work.

#### 4.3. Dynamic Adaptation (self-organization)

Disordered systems act in relation to their environment like living organisms and create a kind of dynamic adaptation between themselves and their environment. Therefore, the self-organization can be defined as a phenomenon in which the system organizes its internal structure independent of external causes.

The intersection of definitions presented for the concept of chaos is stressing on the point that the chaos is the knowledge of behavior investigation of systems that although their input is identifiable and measurable, but the output of this system is random.

Maybe that was why Stuart, the outstanding mathematician, considered this subject as a probability concept, but he was soon amended its definition and reached to a definition that was generally accepted.

According to this definition, chaos is the ability of a simple pattern which does not have any evidence of random phenomena in itself, but it can lead to the emergence of highly irregular behavior in the environment.

### 5. Chaos in Architecture

In general, researchers are examining the phenomenon of chaos in various fields of science, and it has become a common mode. But its impact on art and architecture has not been in the form of science, but as a designing method. Some of the artists from the 1960s created works that are seemingly random, but follow very complicated rules that even the artist himself may not be aware of it.

Chaos concepts, such as non-linearity, doubling frequency and feedback periods, have been increasingly developed in post-modern architecture. According to Charles

Jencks, the architect and master of postmodern architecture of chaos and complexity create these ideas "architecture and torsion waves, the architecture that constantly and unexpectedly swings, grows and decreases.

From our perspective, the chaos is not limited to post-modern architecture. Some traditional structures exhibit the same ideas



*The Bavinger House (Bruce Goff)*

## 6. Fractals

In colloquial usage, a fractal is a multi-sections or uneven geometric shape that can be divided into parts, each of which (at least approximately) a reduced-size copy of the whole form. Benoit Mandelbrot coined the term in 1975 and is derived from the Latin word fractus meaning broken or fractured.

A fractal as a geometric shape generally has the following characteristics:

- Has an elegant structure at optimum small scales.

as well. For example, in the baroque opera building in Paris, which was designed (1825) and built by Charles Garnier between 1861 and 1875, fractional scales can be seen. This building is the amalgam of styles that are based on admissions coordinator. When you walk on Avenue de l'Opéra, details of the building gradually become clear: The closer you are, more detail are seen.



- It is more irregular than to be simply described by traditional language of Euclidean geometry
- It is self-similar (at least approximately or randomly).
- It has a Hausdorff dimension which is larger than its place dimension (topological) (although this condition is not met by space sparse curve like Hilbert curve).
- It has a simple and recursive definition

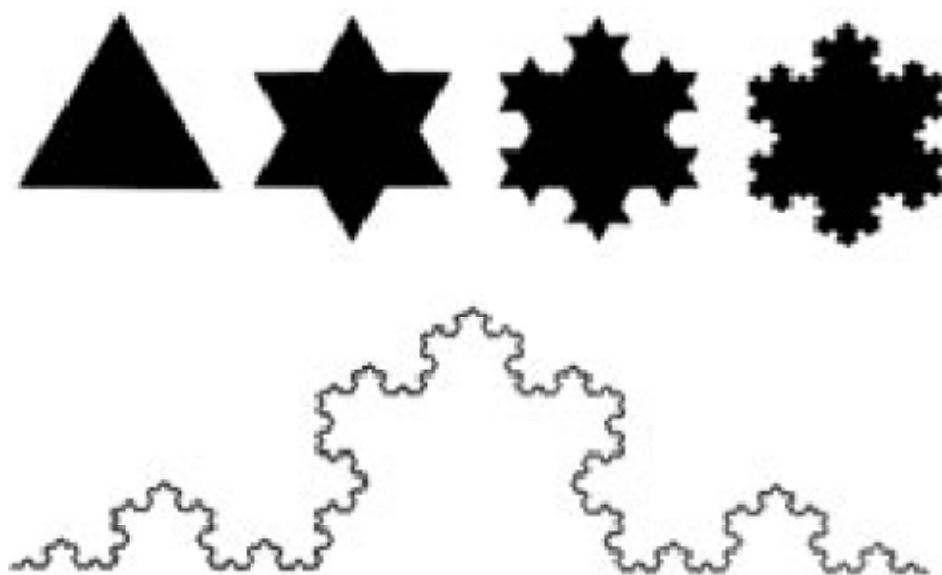
Since fractals are apparent in all levels of magnification, it is assumed that they are indefinitely complex (in informal terms). natural objects which are approximated somewhat to fractals include clouds, mountain ranges, lightning bolts, coastlines and snow. However, all self-similar objects are not fractals, for example, the real line (a straight Euclidean line) is apparently self-similar, but it doesn't have other fractal characteristics.

### 7. Fractal Geometry

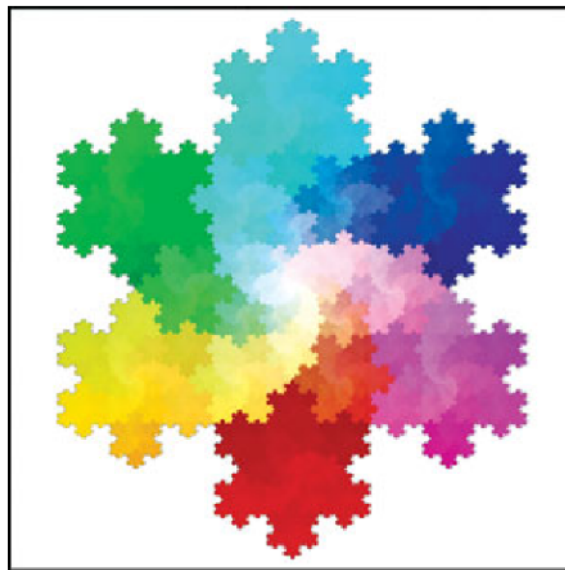
Fractal is a geometric shape or pattern made of the same parts which in the back to the detail reflects the general pattern. In other words, if we look at each component of the

object, the image of the object is created in our minds. The term fractal was coined by Benoit Mandelbrot in 1975 to describe complex geometric objects which has a high degree of self-similarity.

One of the most famous fractals was coined by the mathematician, "von Koch" in 1904. In this fractal called "Koch snowflake" first, we consider an equilateral triangle and divide each side of it into three parts, then instead of the middle of each side segment, we replace an equilateral triangle and we repeat this procedure several times. This type of fractals are called "self-similar" fractals, because every part of it is similar to the larger pieces.

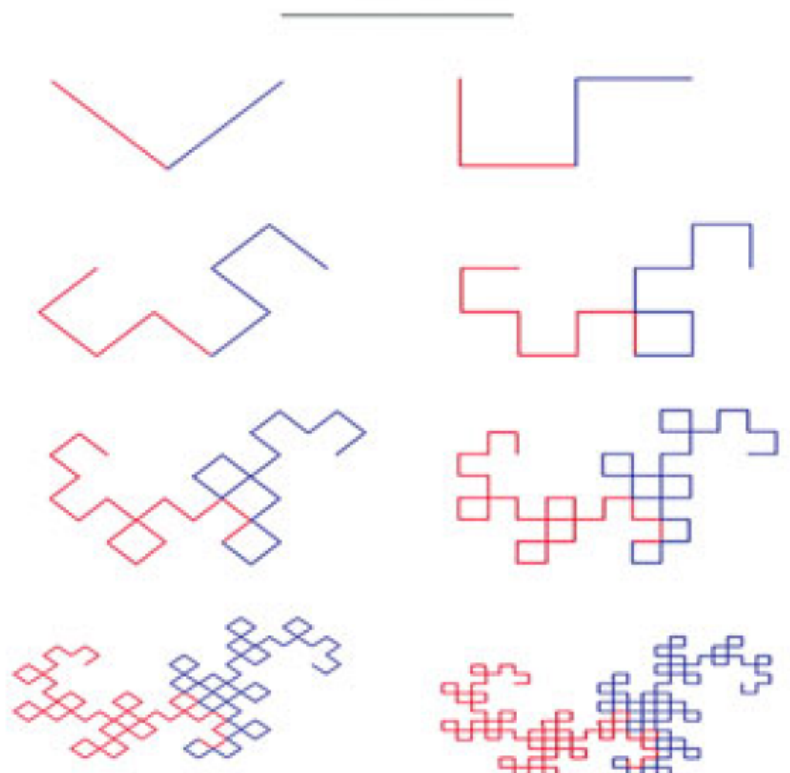


The method of making "Koch snowflake" fractal whose smallest component is an equilateral triangle



The magnified sample of Koch snowflake fractal

Other Fractal is known as "Herter-Heighway *Dragon*" and, unlike its strange name, it has a simple manufacturing method! The figure is formed of a piece of straight line and repetition and pasting them at 90 degrees to each other.



Herter-Heighway *Dragon* making table. see how the figure is repeated and enlarged.

If you look at the table of building this fractal, you see that each house is emerged from the repetition of previous house. It should be said that in many fractals, enlargement process follows special mathematical instructions, each of which are placed in a separate classification.

One of the important, yet complex features of fractals is that the fraction or decimal dimensions are valid. As you know the point doesn't have a dimension, and the line has one and the page has two and volumes have three dimensions.

But in fractal geometry, the discussion is about shapes that have fractional dimensions. For example, if we crumple a sheet of paper (with a thickness close to zero), the result will be a volume which has a decimal dimension.

Fractals are shapes that are not organized in any way unlike forms of Euclidean geometry. First, these shapes are irregular. Secondly, the level of their disorder is not same at all scales.

### 8. The Generation of Fractal Forms

Fractal forms are usually produced using recursive function. For example, recursive functions  $f(n) = f(n) * f(n) + c$  or  $f(n) = f^2(n) + c$  are fractal functions. This equation forms especially a known fractal called the Julia set. In this equation,  $c$  is a

complex number (including an imaginary number) that can be any value, and the result is a different set of Julia.  $n$  will be placed instead of the coordinates of the point. These coordinates are special because, as you guessed it contains imaginary numbers. When the coordinates are  $(x, y)$ , in fractal geometry are shown as  $x + iy$ . In other words,  $x$  is the constant and  $y$  represent the real numbers and the  $y$ -axis represents the imaginary numbers. Now, we will return to the fractal function. We use coordinates  $x + y$  instead of  $n$ . Now you ask how this function makes big fractal charts. Well, the result of a function, instead of showing a line, only shows one point - that if we look at the definition of a point, it can be infinitely small - which expresses how we can enlarge a part of the fractal and reach a new complete fractal. the point is in  $N$  coordinates. Of course, fractals are very colorful. Now how these colors are selected Like everything else, it is relatively simple. First, you need to color a spot, let consider point  $(2 + 1i)$ . For the value of  $c$ , we use  $(1 + 1i)$ . Remember that  $c$  can be any complex number. We put this into the equation now.

$$f(n)=f(2+1i)=(2+1i)(2+1i)(1+1i)=2*2+2i+2i+i^2+1+1i=5+5i-1=4+5i \quad (i^2=-1)$$

- a) They don't take place on the graph (e.g. in a 10 x 10 graph new



components obtained are (97, -234).

- b) Never leave the graph (the law after 200 times repetitions, if the point is still on the charts, it is true.)

The color selection process is in such a way that, if the point leaves the graph after one iteration, we assign a color to it. Any point after that, which leaves the graph after one iteration has the same color. All points leave after 2 iterations are shown with a specific color appear and points which never leave the graph are marked with a distinct color, usually black. After conducting this process, for all parts of the graph, the result will be as same as Julia set.

The function  $f(x) = f^2(x-1) + c$  makes another fractal called Mandelbrot set.

As you can see, in many cases, 200 iterations are needed to determine only one point. In most computers, the number of points for a fractal is 200, 303. That is why to calculate their numerous operations and their accuracy we need computers. Fractals have an image of a real life. Computers can take a real shape or doing repetitions can make them surrealistic. These days, fractals are seen as an important tool in computer graphics, but the time of the emergence of this new concept has played the most important role in

video files compression. These are our new coordinates. Remember that if you put a set of coordinates in a function, the result will be a new set of coordinates.  $4 + 5i$  is a set of new coordinates. The work has not finished yet, the above action shows a repetition.

## 9. Fractal Dimension

One of the very interesting points in the study of fractals is their dimension. For example, we know that the square is a two-dimensional mathematical object. The second dimension can be achieved in such a way that form dividing each side of the square into  $N$  equal parts and connecting the opposite parts together, square  $N_2$  is obtained that the size of each is  $\frac{1}{N_2}$  the first square area. This figure has a fractal structure, each side of the small squares is converted to the main square with  $N$  coefficient. So the dimension of each object can be defined as the ratio of the logarithm of the number of self-similar shapes to magnification factor log.

$$\frac{\log N_2}{\log N_1} = D$$

If you do the same with the Cantor set, since we can make a Cantor set by two Cantor sets with 3 times length, we have:

$$D = \frac{\log 2}{\log 3} = 0.631$$

That is, a Cantor set, is 631/1000 stock. Now, if we look at Fractal Koch shape, we see that

this set is not a complete line that have one dimension and not a point that have no point, but it is the stock between them. For Koch fractal which is more than the line (dimension 1) and less than the page (dimension 2), we have:

$$D = \frac{\log 4}{\log 3} = 1.262$$

Or for SirPinski fractal which covers more space than Koch fractal, but is not reach to a complete list:

$$D = \frac{\log 3}{\log 2} = 1.58$$

In fractals, this fractal dimension matters not the scale. Because in all sizes, the fractal dimension is preserved, and it represents the original property of the fractal.

Fractal geometry is a formal study of self-similarity structure and understanding the concept of complexity in nature (Bovill, 2000: 71) Fractal geometry has self-similarity property and complexity in micro scale and its dimension is not a correct number. Where classical geometry deals with dimensions and correct numbers, the fractal geometry describes dimensions and incorrect numbers.

fractal dimension is not an integer, fractal dimension is placed between 1 and 2. This is in contrast with the Euclidean geometry. The figure of clouds, mountains and coastlines

seem irregular, however, they enjoy a regular geometry whose arrangement is dependent on the scale. If we analyze their structure with different scales, we see always a basic scale in all of them. Fractal dimension is an expression of the relationship between them with different scales. There are several ways to measure the fractal dimension, two of which are described below.

- Self-similarity buildings whose structure components are repeated with different scales
- Fractal dimension with the box – Counting dimension determine fractal dimension of buildings

## 10. Fractal Architecture Features

### 10.1. The Influence of the Surrounding

Society, the natural environment, and the place we live have distinctions; however, we always look at it like a single system. Because in all parts, in some parts of which, we always find similarities that Charles Jencks calls them the organized phenomenon in depth.

In a society based on agriculture, not only its agricultural sector is adapted with agriculture, but its lifestyle and homes have been affected by it. This situation is a reflection of the environment. Fractal

dimension influence from the environment can be investigated on these three items.

### **10.2. Materials**

By magnifying, fractal infrastructure become evident and it is because its structure is not empty of the scale. This is the opposite of simple geometry which is endowed of a defined structure. One of the important characteristics of fractals is that they produce a large number of rules for the distribution of infrastructure.

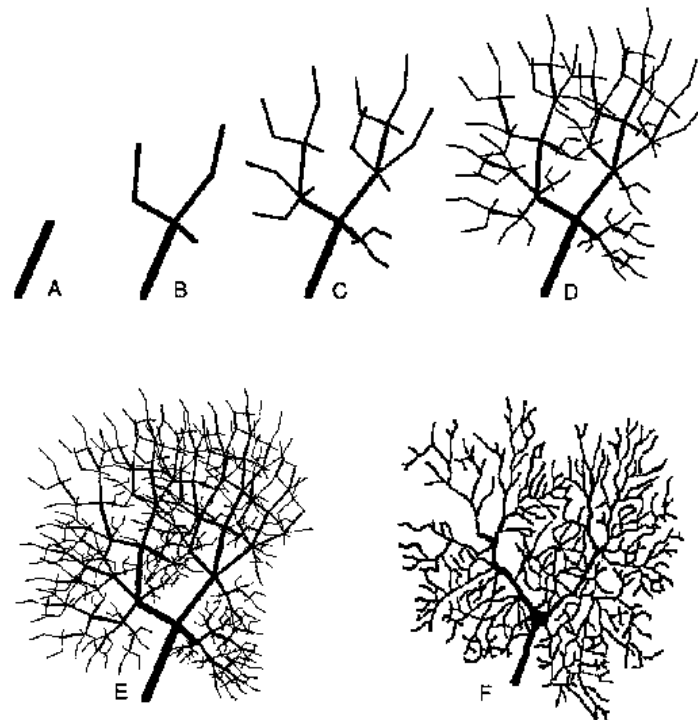
Most of local buildings have made of part of nature or using available building materials and their materials is clearly placed on surfaces with conical features, rugged and like their environment. These houses have a specific similarity with the environment that is repeated through the shape of the building and irregular unevenness in detail. In a vernacular architecture, the materials of different parts have same characteristic and structure.

### **10.3. Landscape and Building**

A certain area, such as buildings, has its own specific fractal dimension. Fractal dimension can answer this question whether particular building or a group of houses are suitable or not. An analysis was done by Karl Boyle created the belief that the harmony between artifacts such as buildings and surrounding environment can be indicated with a fractal dimension. If the environment is very uneven and building were available using nativism and materials creates a high fractal dimension.

### **11. Fractal Structures**

Many examples of fractals can be seen In the nature. Trees, mountains, rivers, beaches, ferns, flowering cabbage are fractal objects. A small part of the tree called branch that is similar to the whole tree. This example can be stated in the clouds, cauliflower, lightning and other objects as a fractal.



FRACTAL PURKINJE MODEL

GUINEA PIG PURKINJE CELL

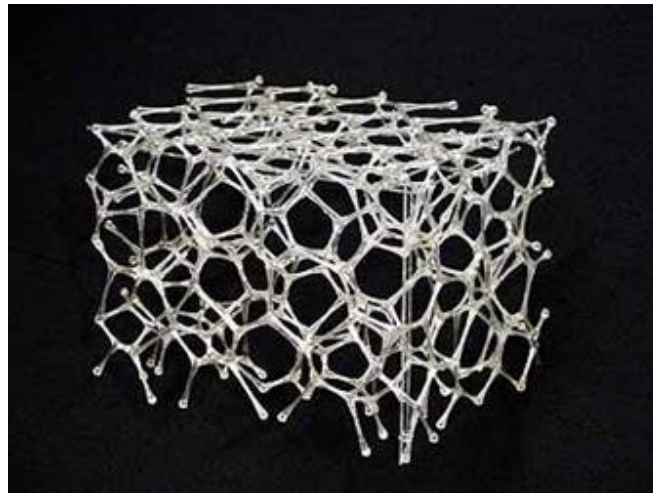
Charles Jencks published in 1995 a book called *Cosmic Jump architecture* based on new scientific findings about the evolution of the universe and its evolution, and non-linear. Jencks explained the genesis of the universe process in simple language in this book. And he stated that the universe is not developed and evolved in a linear form and in one direction, but this evolution has been done in a linear expansion and then it has reached the crisis and chaos border and finally a jump has been done to the past.

Complexity is a theory that says how emerging organisms get rid of balance state in terms of mutual interaction of its components together (due to increased

energy, material or information) and it reaches the boundary between order and chaos. The border is where the system often jumps, breaks or creatively makes a mutual effect. This is done in a non-linear and unpredictable way. New organism may be maintained through the reaction and giving energy continuously. Therefore, in a live and organic universe, organisms and various phenomena are created. And in a linear expansion reach the chaos border and ultimately jump to the organism or something different from previous state. In this case, the process of metamorphosis in butterflies and insects can be stated as examples. For example, butterfly caterpillars

that has a complete metamorphosis process seem as an organism. First it appears as the egg, then the cream, and then a butterfly cocoon, and finally butterfly which each

stage is different from the previous stage. And in each stage of jumping, it creates transformation in the organism.



The pictures of the formation idea of the National Swimming Centre in Beijing



Water Cube, PTW Architects Group, Evo Arup Company

This name is intended for the 2008 Beijing Olympics swimming and water games. The project was considered in the form of a very simple cube, but complex which tried to liken it to the community water molecules. The dimensions of this large cube is 177 \* 177 cubic meters and its height is 31 meters.

This kind of structure is inspired from organic shape of soap bubbles and foam, though this type of structure is frequently observed in nature. In living cells, in crystals forming the material .... and this structure was proposed by two scientists named "phelan" & "weaire" for space division. One

14-sided (including a 12-sided and two regular 6-sided) and the other a 12-sided (including 12 regular five-sided). This solution is currently the best method to divide the three-dimensional space, and despite its sophisticated appearance, it has a superior repeatability. In this type of geometry, only three types of surface, four types of edges and three types of corners (nodes) have been used. To explain natural phenomena, Euclidean geometry does not always apply. In most cases, another type of geometry must be used to show natural side-effects and forms. Fractals are the best language to express this phenomena that can be found even in the architecture. One of the buildings has enjoyed this type of geometry is "Swimming Centre and the Olympic water Games in China 2008," that we have chosen for structural analysis.

## **12. CONCLUSION**

To regulate the seemingly chaotic structure, we need fractal geometry. The fractal geometry not only explains how the simple elements of local order are formed, but it also show how the complexity is created. The study of these concepts, rich and unexpected pattern of tissue structure gives us a new perspective on how architecture is formed and how can we develop organic and harmonious buildings with the environment.

We can propose patterns for reconstruction and development with Fractal dimension and rhythm. Now, the lack of harmony between newly constructed buildings with the local context is considered as one of cities development problems. But, with the modeling we do with the help of fractal dimension, we can obtain a secret order of the context (infrastructure order - self-similarity of coherence) and we can respond to the issue whether the building is designed appropriate to the context or not.

mathematics can be used to understand the fractal patterns and their relationship to adjacent context. In general, it can be said that regarding all man-made organic designs had fractal shape( new architecture ), paying attention to this factor and its evaluation in urban settlements, could be more successful and consistent with the behavior of users, the recognition of spatial texture in modern architecture, accessing to irregular architecture analysis approach modern.

In order to explain the role of mathematics and its application in Iranian architecture, we make the following recommendations:

1. The establishment of community centers among architects and mathematicians for familiarizing with advanced mathematics and its applications in architecture

2. Patterns that are used for the development of cities, if they are compared with the dynamics of fractal geometry and its continuity in small and large scales, some patterns will be generated on this basis which are alive and reliable.

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